

A Programming Language Embedded in *Magic: The Gathering*

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Abstract

Previous work demonstrated that the trading card game *Magic: The Gathering* is Turing complete, by embedding a universal Turing machine inside the game. However, this is extremely hard to program, and known programs are extremely inefficient. We demonstrate techniques for disabling *Magic* cards except when certain conditions are met, and use them to build a microcontroller with a versatile programming language embedded within a *Magic* game state. We remove all choices made by players, forcing all player moves except when a program instruction asks a player for input. This demonstrates *Magic* to be at least as complex as any two-player perfect knowledge game, which we demonstrate by supplying sample programs for Nim and the Collatz conjecture embedded in *Magic*. As with previous work, our result applies to how real *Magic* is played, and can be achieved using a tournament-legal deck; but the execution is far faster than previous constructions, generally one cycle of game turns per program instruction.

2012 ACM Subject Classification Theory of computation → Representations of games and their complexity

Keywords and phrases Programming, computability theory, *Magic: the Gathering*, two-player games, tabletop games

Digital Object Identifier 10.4230/LIPIcs.FUN.2024.18

Category FUN with Algorithms

Related Version *Full Version*: https://www.toothycat.net/~hologram/Magic/Magic_Microcontroller_full.pdf

1 Introduction and Previous Work

Magic: The Gathering (also known as *Magic*) is the world’s largest tabletop collectible card game, played by hundreds of thousands of players in tournaments and by millions more players casually. In 2020, Churchill, Biderman & Herrick published an embedding of a universal Turing machine inside *Magic* [1]. This is the first widely played tabletop game to be shown Turing complete in the format in which it is usually played, as opposed to some infinite generalisation. For example, chess is EXPTIME-complete with infinite board and pieces, but has a finite number of states in the 8x8 board used for tournament play. Churchill et al. showed that the question “will this *Magic* game ever terminate” cannot be answered in the general case, even for two-player *Magic* played with all the usual tournament restrictions. However, this paper did not contain any concrete example computations.

The author of [2] investigated the runtime performance of this Universal Turing Machine (UTM) embedded within *Magic*. He established a compilation sequence from an arbitrary Turing machine with N states, into a 2-tag system, into the 2-state UTM(2,18), and thence into *Magic*. He supplied a simple Turing machine to compute $2+3$ in a unary adder. However, he found that no simulation was able to establish how long it would take to compute $2+3$ in the UTM. The straightforward compiler’s output from this simple Turing machine for



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12th International Conference on Fun with Algorithms (FUN 2024).

Editors: Andrei Z. Broder and Tami Tamir; Article No. 18; pp. 18:1–18:37



Leibniz International Proceedings in Informatics

Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

computing $2+3$ results in a tape over 40 million symbols long (15 million in the program and 25 million in the data). The UTM simulation of the Turing machine needs to constantly move between the program and data sections, resulting in absurdly inefficient computation times. No simulation was able to establish how long it would take to compute $2+3$ in the *Magic* game.

The same author created an optimised unary adder Turing machine with just 2 states, and compiled a simplified computation of $1+1$. This completed in a mere 3,958,876,878 game cycles (of each of the two players taking a turn). Even after nearly 8 billion game turns, because of the multiple steps in translating the calculation into the UTM(2,18), the output consisted of hundreds of creature tokens, which needed to be interpreted by carefully counting how many tokens of type Myr were mixed in among the hundreds of tokens with type Aetherborn, all in order to retrieve the output of “2”.

In this paper we set out a different construction, which embeds a full microcontroller within *Magic: The Gathering*. In Section 2 we describe at a high level some key features of the construction, and Section 3 specifies the programming language supported by the microcontroller. Section 4 and Appendix B provide the precise details of how all this is implemented, and we work through an example instruction in Section 5. Section 6 contains our conclusions and discussion of the implications. Finally, Appendix A provides several sample programs, Appendix C explains which cards we use to modify other cards, and Appendix D supplies a decklist that could be brought to a *Magic* tournament to assemble the construction.

2 Outline of the Construction

The rest of this paper describes a construction that simulates a fully general programming language within *Magic: The Gathering*. Compared to [1] this is also Turing complete, but is much more efficient and easy to program, and reports its outputs much more clearly. It also allows reading input from each of the players of the two-player *Magic* game in which it is embedded, and can be programmed to terminate in a win for either player or a draw, or of course can keep running indefinitely.

As with the Turing machine construction, we start by assuming one player, Alice, draws a combination of cards that allows her to take over the game, draw all the rest of her deck, and remove all cards from the hand of the opponent, Bob. After the initial setup is completed, she removes all player choices, so that neither player has any option but to let the program execution continue, short of conceding the game. Thus the outcome of this tournament-legal *Magic* game is entirely determined by the result of the program.

The program is written in a language of 12 symbols, represented by basic land cards, which are allowed to occur any number of times in a player’s deck. Alice’s deck needs to start off containing a lot of other cards, but once the microcontroller is set up and she has drawn all her cards, she returns to her deck a sequence of cards from among these 12, which encodes the program to be executed.

The program is read one card at a time. During Alice’s combat step we put one of these cards, which we call the “program permanent”, onto the battlefield for long enough to read it, then move it to the bottom of Alice’s deck. We in fact make Alice have three combat steps each turn, and in each one, a program permanent is read and possibly other game actions are (automatically) taken. Each instruction in the programming language is a sequence of three symbols, interpreted by some set of “instruction permanents” that do extra things during Alice’s or Bob’s turn.

All instruction permanents that don't apply to the current instruction are "inactive", i.e. have all their abilities removed. To do this we turn them into creatures with a certain creature type (e.g. Angel), have a card which makes all Angels also Saprolings, make all Saprolings lands, and remove abilities from all lands. Crucially, we can conditionally allow a certain group of instruction permanents to regain their abilities by temporarily "phasing out" the card which makes (e.g.) Angels into Saprolings. (Objects in *Magic* which are "phased out" are treated by the rules as if they don't exist.) We do this during one of Alice's combat steps, so those instruction permanents have their abilities during the rest of Alice's turn (including her later combat steps) and all of the other player Bob's turn.

Permanents in *Magic* have zero or more colours, from the set of white, blue, black, red and green. Permanents that are creatures have zero or more creature types, also drawn from a well-defined set, but this set has over 200 types in it. We use both of these characteristics extensively. All "inactive" permanents are made green Saproling creatures, and all green creatures are given protection from certain creature types. We make extensive use of the capabilities *Magic* offers to edit existing cards by changing colour words (using the card **Mind Bend**), creature type text (using **Artificial Evolution**), colours (using **Prismatic Lace**), and creature types (using a variety of cards according to circumstances).

In particular, we carefully apply restrictions so that any time an ability triggers that would normally let its controller choose a target, there is precisely one legal target (or occasionally no targets, which prevents the ability from going on the stack at all). We ensure that all creatures attack and block where possible using **Grand Melee**, but make most creatures unable to attack or block using **Stormtide Leviathan**. Any time a creature is able (and thus forced) to attack, we arrange that either it can't be blocked at all, or there is precisely one creature forced to block it.

3 The Programming Language

The programming language we implement has the following features:

- Twelve registers $r_0 \dots r_{11}$, each able to contain an arbitrarily large nonnegative integer.
- An unlimited number of memory slots, each addressed by a nonnegative integer address; each memory slot can hold a single arbitrarily large nonnegative integer.
- A single Boolean flag that is set by certain instructions such as comparisons. The flag can be read by certain instructions; most notably, jump instructions can be made conditional on whether the flag is true or false.

The program is written in a language of 12 symbols, and each instruction is a sequence of three symbols. For example, the sequence 0 1 2 (represented by cards Plains Island Swamp) encodes the instruction "Add 1 2", which will result in increasing the value of register r_1 by the value of r_2 . We provide the following instructions in the language:

| | | |
|----------------------|-------------------|--|
| 5 Y Z ($Y \neq Z$) | Set $r_Y r_Z$ | Set r_Y to the value of r_Z . |
| 5 Y Y | Zero r_Y | Set r_Y to zero. |
| 0 Y Z | Add $r_Y r_Z$ | Set r_Y to $r_Y + r_Z$. |
| 4 1 Z | Add1 r_Z | Set r_Z to $r_Z + 1$. |
| 4 2 Z | Halve r_Z | Set r_Z to half the value of r_Z , rounding down. Set the flag to the remainder from the division. |
| 1 Y Z ($Y \neq Z$) | SubCond $r_Z r_Y$ | If $r_Z \geq r_Y$, set r_Z to $r_Z - r_Y$ and set the flag to 0. Otherwise, set the flag to 1. |

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| | | | |
|----|------|-------------------|---|
| 1 | Z Z | Sub1Cond r_Z | If $r_Z \geq 1$, set r_Z to $r_Z - 1$ and set the flag to 0. Otherwise, set the flag to 1. |
| 6 | Y Z | Mult $r_Z r_Y$ | Set r_Z (note, not r_Y) to $r_Y \times r_Z$. |
| 7 | Y Z | DivCeil $r_Y r_Z$ | Set r_Y to $\lceil r_Y/r_Z \rceil$. If the division was exact, set the flag to 0, otherwise set it to 1. If $r_Z = 0$ or $Y = Z$, this is undefined behaviour. |
| 2 | Y 0 | AInput r_Y | Set r_Y to a nonnegative integer of Alice's choice. |
| 2 | Y 1 | BInput r_Y | Set r_Y to a nonnegative integer of Bob's choice. |
| 2 | Y 2 | SetF r_Y | Set r_Y to the flag's value. |
| 2 | Y 3 | SetNF r_Y | Set r_Y to the Boolean negation of the flag's value (1 if it's 0 and vice versa). |
| 2 | Y 4 | Rand6 r_Y | Set r_Y to a random nonnegative integer less than 6. |
| 2 | Y 5 | Rand20 r_Y | Set r_Y to a random nonnegative integer less than 20. |
| 10 | Y Z | NumBuild $12Y+Z$ | Set r_0 to $12Y+Z$, except if the last instruction that was executed was also a NumBuild instruction, in which case multiply r_0 by 144 and add $12Y+Z$ to it. As its name suggests, this instruction can be used repeatedly to build any nonnegative integer value in r_0 , two base-12 digits at a time. |
| 8 | Y Z | Store $r_Z r_Y$ | Store the value of r_Y at memory address r_Z . |
| 9 | Y Z | Load $r_Y r_Z$ | Load the value at memory address r_Z into r_Y . |
| 3 | 0 Z' | JumpFwd Z' | Jump forward by Z' instructions. If $Z = 0$, Z' is r_0 , and otherwise, Z' is Z . Thus instead of a useless command to jump 0 instructions, we gain the ability to jump an arbitrary or computed distance. |
| 3 | 1 Z' | JumpBwd Z' | Jump backward by Z' instructions. Backwards jumps by more than the length of the program ¹ do nothing. |
| 3 | 2 Z | JumpFwdNF Z' | Jump forward by Z' instructions if the flag is 0/false. |
| 3 | 3 Z | JumpBwdNF Z' | Jump backward by Z' instructions if the flag is 0/false. |
| 3 | 4 Z | JumpFwdF Z' | Jump forward by Z' instructions if the flag is 1/true. |
| 3 | 5 Z | JumpBwdF Z' | Jump backward by Z' instructions if the flag is 1/true. |
| 3 | 6 0 | CallFwd r_0 | Call a function r_0 instructions ahead: Jump forward $3r_0$ cards and push $P - 3r_0$ onto the return stack. P is the length of the program in cards ¹ . If $r_0 = 0$ or $3r_0 > P$, this is undefined behaviour. |
| 3 | 7 Z | Return Z' | Return from a function Z' instructions long: Pop a value S from the return stack, and jump forward $\max(0, S - 3Z')$ cards. If the return stack was empty, end the game in a draw. |
| 3 | 6 1 | CallBwd r_0 | Call a function r_0 instructions behind: Jump backward $3r_0$ cards and push $3r_0$ onto the return stack. If $3r_0 \geq P$, this is undefined behaviour. May not be used to call a function from within itself. |

¹ If the program is less than 6 cards long, P is the first multiple of the length that is at least 6.

| | | |
|-----------------------|-----------------|--|
| 3 6 2 | CallBwdR r_0 | Call a function r_0 instructions behind (direct-recursion-capable): Same as CallBwd, but push $P + 3r_0$, so that this may be used to call a function from within itself. |
| 11 Y Z ($Y \neq Z$) | FLess $r_Z r_Y$ | Set the flag to 1 if $r_Z < r_Y$, 0 otherwise. Flag-combining. |
| 11 Z Z | FIsZero r_Z | Set the flag to 1 if $r_Z = 0$, 0 otherwise. Flag-combining. |
| 4 0 0 | HaltD | End the game in a draw. |
| 4 0 1 | HaltA | End the game with Alice winning. |
| 4 0 2 | HaltB | End the game with Bob winning. |

“Flag-combining” is a property used by some instructions whose only purpose is to set the flag. It means that flag values given by subsequent instructions are combined by logical OR instead of replacing the flag’s value. This state ends when any of the following is executed:

- Any instruction that uses the flag’s value.
- Any jump instruction (including calls and returns, and regardless of whether the jump is taken or not).

For example, this can be used to check whether r_0 and r_1 are equal, with FLess $r_0 r_1$ followed by FLess $r_1 r_0$.

The program is cyclic, which is to say it wraps around: after passing the final instruction (by executing it or jumping past it) execution continues with the first instruction. Similarly you can jump backwards beyond the start of the program and end up near the end of the program.

Encountering a sequence that does not match any of the instructions above is undefined behaviour.

These instructions are a superset of those required for a random-access register machine such as Melzak’s Q-machine, which is Turing complete.[3]

4 Implementation of the Microcontroller

In this section we describe the various gadgets that make up the microcontroller as described in the previous section. We defer discussion of how to set up the board state (including modifying card types, creature types, colours, etc) to Appendix C.

Note that this PDF contains tooltips: hover over a bold card name to see its full rules text, and hover over a dashed underline to see how specific modifications are accomplished or reminders of how creature types are used.²

4.1 The program

The program is a sequence of the following cards: **Plains**, **Island**, **Swamp**, **Mountain**, **Forest**, **Wastes**, **Snow-Covered Plains**, **Snow-Covered Island**, **Snow-Covered**

² These display in Adobe Reader but may not display in web browsers. Readers using a web browser may like to read the details at [4] instead, where we provide more detailed tooltips.

Swamp, Snow-Covered Mountain, Snow-Covered Forest, and Snow-Covered Wastes³. We call these cards “symbol cards” and assign them numbers 0, 1, 2, . . . , 11 in the order listed.

Most of the time, one of the cards in the program will be on the battlefield under Alice’s control; we call this card the *program permanent*. The rest of the cards will be in Alice’s library, with the next symbol in the program on top of the library, continuing in program order from top to bottom, then continuing from the start of the program until the symbol before the current symbol.

The program is made up of instructions that each consist of 3 cards. X, Y, and Z shall refer to the numbers of the three cards that form the instruction currently being executed, in that order. For example, the instruction “Add r_1 r_2 ” is represented by 0 1 2 (Plains Island Swamp), and X is 0, Y is 1 and Z is 2.

4.2 Global environment control

On the battlefield is a **Grand Melee** (“All creatures attack each combat if able. All creatures block each combat if able”) and a **Stormtide Leviathan** (“Creatures without flying or islandwalk can’t attack”). Most creatures are unable to attack; when we want a creature to be able to attack, we will give it flying or islandwalk. Bob’s creatures with islandwalk will be unblockable because Alice controls the program permanent which is given type Island.

Both players have life total 1, and each has a **Worship** keeping their life totals at 1 through the damage they will be dealt. (Note that **Worship** only modifies the *result* of damage; the damage itself is still dealt, so effects triggered on combat damage still trigger.)

4.3 Advancing through the program

Alice has a **Vaevictis Asmadi, the Dire**, whose rules text reads “Whenever Vaevictis Asmadi, the Dire attacks, for each player, choose target permanent that player controls. Those players sacrifice those permanents. Each player who sacrificed a permanent this way reveals the top card of their library, then puts it onto the battlefield if it’s a permanent card.”

We use the techniques in Appendix C to make **Vaevictis Asmadi, the Dire** into a 1/1 **Sliver Beast Reflection**. It attacks in Alice’s combat phase. Its ability is forced (see subsection 4.6) to target the program permanent for Alice, and one particular permanent for Bob. When it resolves, Alice sacrifices the program permanent, and **Wheel of Sun and Moon** enchanting Alice sends it to the bottom of Alice’s library; then the next card (on top of Alice’s library) is put onto the battlefield, becoming the new program permanent. Meanwhile, Bob has been given a **Tajuru Preserver** stopping Bob’s permanent from being sacrificed, and thus Bob does not reveal the top card of his library.

Alice controls **Tetsuko Umezawa, Fugitive** ensuring that **Vaevictis Asmadi, the Dire**, and all her other creatures with power or toughness 1 or less, can’t be blocked.

4.4 Disabling and conditionally enabling permanents

For each of the 12 program cards, letting n be its number, we choose a creature type X_n . X_0 is Aetherborn, X_1 is Beeble, and so on through Cephalid, Drake, Eldrazi, Faerie, Gremlin, Homarid, Illusion, Juggernaut, Kavu, and Lhurgoyf.

³ To be released on 7th June 2024. To play the Microcontroller before that date, instead use Persistent Petitioners and Infinite Reflection as described at [4].

We have Alice control twelve token copies of **Dralnu’s Crusade**, whose printed rules text reads “All Goblins get +1/+1. All Goblins are black and are Zombies in addition to their other creature types.” However we use the techniques in Appendix C to edit each one to instead to affect creatures with a different one of the 12 creature types X_n ($n \in \{0, 1, \dots, 11\}$), and add type Saproling. **Life and Limb** makes Saprolings into lands, and also makes them green, and then **Blood Sun** removes their abilities. So all creatures with any of the 12 creature types X_i have their abilities removed.

Each of these **Dralnu’s Crusades** is made into a creature with creature type Sliver, with power and toughness 2/1, and given flying. For each of the 12 basic land cards, the corresponding **Dralnu’s Crusade** is equipped with a **Strata Scythe** imprinted with that basic land card (“Equipped creature gets +1/+1 for each land on the battlefield with the same name as the exiled card”).

All twelve **Dralnu’s Crusades** are forced to attack, and the one corresponding to the current program permanent will be boosted to 3/2; the previously mentioned **Tetsuko Umezawa, Fugitive** ensures that the others can’t be blocked.

Bob controls a **Dream Fighter** (“Whenever Dream Fighter blocks or becomes blocked by a creature, Dream Fighter and that creature phase out”) with creature type Sliver and granted reach (so it can block creatures with flying). This blocks the 3/2 **Dralnu’s Crusade**, and they both phase out, so that creatures of the type corresponding to the current program permanent are no longer made Saprolings. We have a **Shadow Sliver** ensuring that only Slivers can block these **Dralnu’s Crusades**.



■ **Figure 1** After Vaevictis Asmadi, the Dire puts a Wastes onto the battlefield, the only one of Alice’s attacking **Dralnu’s Crusades** which Bob’s **Dream Fighter** can block is the one given +1/+1 by a **Strata Scythe** imprinted with Wastes.

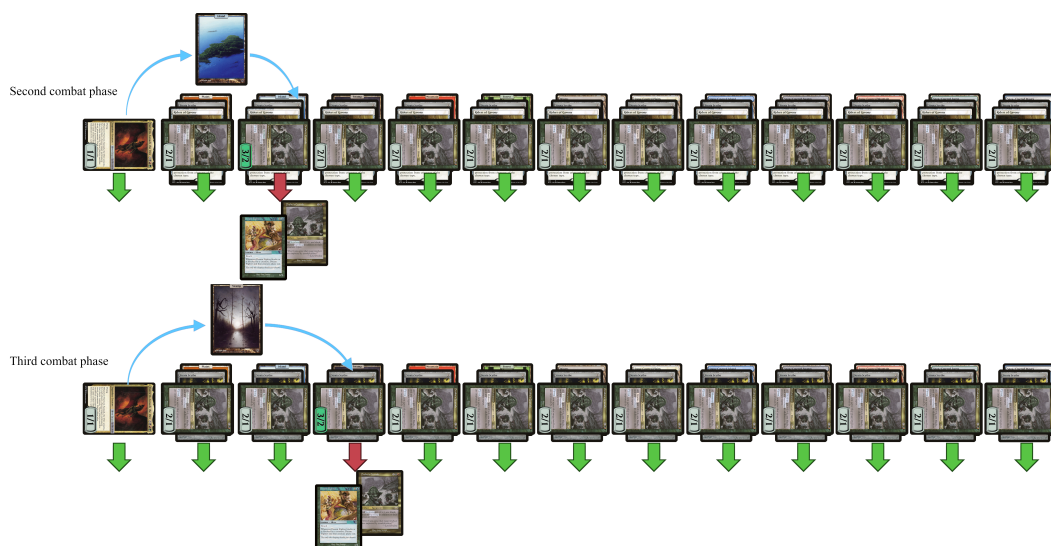
This reads the first card of an instruction; creatures of the corresponding type will regain their abilities after **Dream Fighter’s** ability resolves. We also choose a thirteenth creature type (Monkey) denoted X_A , and have another **Dralnu’s Crusade** making X_A creatures also Saprolings, but this one is not a creature; rather we make it an artifact so **Bludgeon Brawl** makes it an Equipment, and attach it to Bob’s **Dream Fighter**, so it will phase out when the **Dream Fighter** does. This means that creatures of type X_A get to regain their abilities after the first card is read, no matter what it is, but they will lose their abilities again when Bob’s turn starts and the **Dream Fighter** phases back in.

We will also often want to use this conditional mechanism on noncreature permanents; to do that, we make token creature copies of them using **Urza, Prince of Kroog**, combined with **Memnarch** if necessary.

4.5 Reading the second and third cards

Alice controls a **Bloodthirster** (“Whenever Bloodthirster deals combat damage to a player, untap it. After this combat phase, there is an additional combat phase. Bloodthirster can’t attack a player it has already attacked this turn.”) It is made a 1/1 Sliver Beast Reflection and given double strike. This also attacks, can’t be blocked because of **Tetsuko Umezawa, Fugitive**, and adds a second combat phase. The second combat phase is used to read the second card, using another copy of the above setup, but with all the creatures involved granted an additional creature type of X_A so that they do not attack or block in the first combat phase (because they don’t have their abilities at that point). So there is a second **Vaevictis Asmadi, the Dire** to advance through the program, given types Sliver Beast Reflection X_A ; a second **Dream Fighter** with reach and types Sliver X_A ; and another batch of **Dralnu’s Crusades** for another 13 creature types $Y_0, Y_1, \dots, Y_{11},$ and Y_A (Antelope, Basilisk, Camel, Dauthi, Efreet, Fox, Gnome, Hippo, Inking, Jellyfish, Kor, Lammasu, and Metathran), the first 12 having **Strata Scythes**.

Similarly, since the **Bloodthirster** has double strike, it will also give Alice a third combat phase, which is used by another copy of the setup (with creature type Y_A) to make another 13 creature types $Z_0, Z_1, \dots, Z_{11},$ and Z_A conditional on the third card of the instruction (Aurochs, Brushwagg, Camarid, Druid, Elephant, Ferret, Graveborn, Hamster, Imp, Jackal, Kithkin, Licid, and Masticore).



■ **Figure 2** More Dralnu’s Crusades attack in the second and third combat phases, and another of each is blocked and phases out. They couldn’t attack before because of their types X_A or Y_A .

4.6 Constraining targets

Almost all creatures will be green; any creature that isn’t naturally green and isn’t specified to be differently coloured is made green by **Prismatic Lace**. A few creatures will be red or blue instead, but even those are made green while inactive by **Life and Limb**.

A **Masked Gorgon** edited to give green and blue creatures protection from Reflections means that only red creatures are legal targets for Reflections’ abilities. Similarly, a **Masked Gorgon** edited to give green and red creatures protection from Beasts means that only blue creatures are legal targets for Beasts’ abilities.

We will refer to the creature types Reflection and Beast as tR and tU respectively, indicating their function. (“U” is the usual abbreviation for “blue” in *Magic*.)

As mentioned earlier, each **Vaevictis Asmadi, the Dire** has been made both a Beast and a Reflection, so that green creatures, blue creatures, and red creatures are all illegal targets for it; the intended target under Alice’s control is a noncreature land. The same types are applied to the **Bloodthirster** for a different purpose, to stop it from blocking on Bob’s turn (which is something else protection does), because it untaps itself and stays untapped (as its own ability prevents it attacking in the later combat phases).

Spectral Guardian makes noncreature artifacts illegal targets for anything. Alice and Bob both control a **Sterling Grove** to make other enchantments illegal targets for anything. Alice’s is made an artifact so it gains shroud from **Spectral Guardian**. Bob’s one does not itself have shroud, and thus it is the only legal target under Bob’s control for **Vaevictis Asmadi, the Dire**, but Bob has a **Tajuru Preserver** so he does not sacrifice anything. We do however give Bob’s **Sterling Grove** protection from blue, which will be useful later to stop some other things from targeting it.

Alice and Bob both have **Ivory Mask**, making both players illegal targets for anything.

4.7 Order of continuous effects, and one more of them

The *Magic* Comprehensive Rules [5] specify a system of “layers” for working out what happens when multiple effects apply to the same permanent. For example, effects that make one permanent a copy of another object apply in layer 1. All effects that change a permanent’s type (such as creature, land, etc) or subtype (Angel, Goblin, etc) apply in layer 4. Anything that adds or removes abilities applies in layer 6, and so on. Within a layer, if multiple effects try to affect the same permanent, each object or effect has a “timestamp”, generally when that object or effect was created. Within this document, we denote timestamps with circled numbers: an effect with timestamp ① will take effect earlier than timestamp ②.

The continuous effects mentioned so far are timestamped as follows:

- ①: **Stormtide Leviathan**
- ②: **Dralnu’s Crusades** and **Blood Sun**
- ③: **Life and Limb** and protection-granting and shroud-granting effects

Any other continuous effect is timestamped ① unless otherwise stated.

The exception to timestamp order is “dependency”: if two effects would apply within the same layer, but one will change the existence of the second or which objects the second acts on or what it does to them, the first one applies first even if the second has an earlier timestamp. This applies to our construction where abilities that will be removed by the **Blood Sun** wait for it to be applied (and thus end up not being applied themselves).

The program permanent is a land so it is granted type Island by the **Stormtide Leviathan**, but it may be a Forest as well. We do not want **Life and Limb** to make the program permanent a creature. So we also have Alice control an **Illusionary Terrain**, made a creature with type Z_A , within timestamp ① after the **Stormtide Leviathan**, turning all Islands to Islands. This is not as ineffective as it sounds: rule 305.7 [5] says that this removes all other types, so that the program permanent’s subtype is set to Island and no others. Note that, because there are no Saprolings before ②, the **Stormtide Leviathan** does not have a dependency on the **Life and Limb**.

By being made a Z_A , this **Illusionary Terrain** has its abilities removed by **Blood Sun** most of the time, most importantly during Alice’s upkeep. Because the effect of setting basic lands’ types is applied in layer 4, before the abilities are removed in layer 6, it still

functions despite the abilities being removed. However, the removal of abilities does shut off its cumulative upkeep.

4.8 Registers

There are twelve registers named r_0, r_1, \dots, r_{11} , each of which holds a nonnegative integer value.

Each register r_n is a token copy of **Joraga Warcaller** under Bob’s control, given copiable creature type Z_n and additionally given creature type Rabbit, given indestructible and vigilance, coloured red at timestamp ④, with base power and toughness noncopiously set to 2/2 but with two -1/-0 counters, and a number of +1/+1 counters on it equal to its register value. (As always, see Appendix C or the tooltips for how all these changes are accomplished.) **Joraga Warcaller**’s rules text says “Other Elf creatures you control get +1/+1 for each +1/+1 counter on Joraga Warcaller”. Thus, after Z is read from the program, the active register r_Z regains its ability and adds its value to the power and toughness of each other Elf that Bob controls. (This does not include the other registers; their Elf type is overwritten.)

Note that each register’s power is equal to its value, whether it is r_Z (having base power 2 and two -1/-0 counters) or not (having base power 1 from the **Life and Limb**, +1 power from a **Dralnu’s Crusade**, and two -1/-0 counters).

All the registers are made red by **Prismatic Lace** at timestamp ④, later than that of the **Life and Limb**, so that they are always red even when inactive. But the inactive registers are still Saprolings even though they’re red.

r_0 also has Rhino added to its creature types; this will be useful for some instructions that use specifically this register.



■ **Figure 3** The first three registers. In this case r_0 has value 3, r_1 value 0 and r_2 value 2. If $Z = 0$, the **Dralnu’s Crusade** making Aurochs into Saprolings is phased out and Bob’s Elves get +3/+3.

For each $n \in \{0, 1, \dots, 11\}$, Bob has a **Riders of Gavony** giving Z_n creatures protection from Yetis. Each of these is made into a noncreature artifact with mana value 0 and attached to the **Dralnu’s Crusade** that applies to Y_n , so that they phase out together. As a result, this means that each register except for r_Y has protection from Yetis. Then, a creature can be given the types **Reflection** and **Yeti** so that it can only target r_Y ; we call this combination *try*.

Bob also has another **Riders of Gavony** giving Saprolings protection from Zubera creatures. This means the inactive registers (those other than r_Z) can't be targeted by any creature that's a Zubera. Then, as above, creature types Reflection and Zubera together mean that a creature can only target r_Z ; we call this combination tr_Z .

4.9 Memory

We provide an unlimited number of memory slots, each addressed by a nonnegative integer address. Each memory slot can hold a single arbitrarily large nonnegative integer.

A nonzero value V at a memory address A is represented by a Mouse token with base power and toughness V/V under Alice's control with $A + 1/+1$ counters on it. A zero value is represented by an absence of such a token.

4.10 The flag

There is a Boolean flag that some instructions use. It is represented by a card, being in Bob's library for 0/false and in Bob's hand for 1/true; this card must not have any abilities that function in those zones other than characteristic-defining abilities. We assume that Bob has at least one such card in his deck (most lands, planeswalkers, instants and sorceries would be suitable). We remove all other cards from Bob's hand, library and graveyard.

Wheel of Sun and Moon enchanting Bob allows us to set the flag to 0 by making Bob discard the card. We give Bob a **Tomorrow, Azami's Familiar**, allowing us to set the flag to 1 by making Bob draw a card, while not making Bob lose the game if the flag was already 1.

To stop Bob from playing this card, we give Bob a **Nevermore** and/or an **Aggressive Mining made into an artifact** as appropriate.

4.11 Further environment control

To prevent any player choice involving the program permanent's abilities, we use **Root Maze** to make each new program permanent enter the battlefield tapped and **Choke** to keep them tapped (recall that they are all made Islands). Also, Bob's **Sterling Grove** has its activated ability shut off by **Suppression Bonds** attached to it. **Stony Silence** and **Cursed Totem** shut off activated abilities of artifacts and creatures.

Both players control a copy of **Recycle**, skipping both player's draw steps. **Mirror Gallery** disables the "legend rule". And we give both players a **Corrosive Mentor** so that black creatures controlled by either player have wither.

4.12 Instructions

See Appendix B for details on how the individual instructions in the language are implemented.

5 Example Instruction

For demonstration purposes, here is how an example turn cycle looks. Let us say the next three cards on the top of Alice's library are Wastes, Plains, Swamp. This triplet encodes symbols 5 0 2, a Set instruction.

At the start of Alice's turn, most creatures are Saprolings and therefore have no abilities. Recall that all creatures are forced to attack and block where able, but only creatures with flying or islandwalk are allowed to attack. In Alice's first combat phase, twelve flying 2/1

Dralnu's Crusades attack along with the **Bloodthirster** and **Vaevictis Asmadi, the Dire**, whose ability puts the Wastes onto the battlefield. This makes the **Dralnu's Crusade** affecting X_5 get +1/+1 from its **Strata Scythe**, and so it gets blocked by Bob's first **Dream Fighter** and phases out. Creatures with type X_5 or X_A regain their abilities (unless they also have another type making them a Saproling such as Y_n).

In the second combat phase (granted by **Bloodthirster**), another **Vaevictis Asmadi, the Dire** and twelve more **Dralnu's Crusades** attack, as their type X_A is no longer causing their flying ability to be removed. This Vaevictis's ability puts the Wastes onto the bottom of Alice's library and the next card of the program in its place, the Plains. The **Dralnu's Crusade** affecting Y_0 gets +1/+1 from its **Strata Scythe** and gets blocked by Bob's second **Dream Fighter**, whose type X_A is no longer having its reach ability removed. Creatures with type Y_0 or Y_A regain their abilities.

In the third combat phase, Alice's **Archpriest of Iona** with types $X_5 Y_A tr_Y$ Cleric has finally regained its abilities. Its ability triggers, and is forced to target r_0 , because its types tr_Y mean it can't target any green or blue creatures or any of the other registers. r_0 gains flying, so it'll be able to block, and gets a temporary +1/+1.

When the third set of **Dralnu's Crusades** and the third **Vaevictis Asmadi, the Dire** attack, Alice's **Shape Stealer** with types $X_5 Y_A$ is also forced to attack. The **Dralnu's Crusades** all have shadow, so r_0 can't block any of them; the only creature r_0 can block is the **Shape Stealer**. **Shape Stealer's** ability gives it base power equal to r_0 's value +1, which is why it has the -1/-0 counter so its actual power is r_0 's value. It is given wither because it is black, so the damage is dealt as -1/-1 counters, cancelling out all the +1/+1 counters on the register and setting r_0 's value to 0.



■ **Figure 4** The five steps of instruction 5 0 2, Set $r_0 r_2$.

In Bob’s combat phase, **Halana and Alena, Partners** triggers. Because it is an **Elf**, its power is equal to r_2 ’s value. And because it has types t_{r_Y} as well, just like with Alice’s **Archpriest of Iona**, the only legal target for its trigger is r_0 . So it adds $r_2 +1/+1$ counters to r_Y . Then nothing else happens on the rest of Bob’s turn, and we move back to Alice’s turn, when the three copies of **Vaevictis Asmadi, the Dire** will read three more cards from the program.

6 Implications and Conclusion

6.1 Readability and programmability

In sharp contrast to the impenetrable millions of tokens produced by the Turing machine in [1], the game state will be clearly readable when a program in this construction terminates. After the computation of sample program 1 “Calculate 10 cubed” (see Appendix A), there will be one Mouse creature token with power and toughness 1000/1000. When sample program 2 “Prime Factors” halts, for each prime factor of the input number, there will be one Mouse creature token with power and toughness equal to that factor. When sample program 4 “Nim” halts, the result of the *Magic* game will be victory for Alice or Bob according to who won the embedded game of Nim.

The programming language provided is comparable to other microcode programming languages and assembly languages. It has some quirks but is perfectly usable to write moderate-sized programs. Readers are invited to write their own programs in the simulator we wrote to test the sample programs [6].

6.2 Tournament playability

The construction uses many different *Magic* cards, far more than are normally included in tournament decks. But it is legal to bring a deck with more than 60 cards to a tournament; players sometimes play decks with over 200 cards [7]. The only restriction is that you must be able to physically shuffle the deck in a reasonable amount of time [8].

Appendix D contains a decklist of a 360-card deck which could be brought to a Legacy tournament. The deck’s composition breaks down as 160 land cards to be used for the program; 136 distinct named cards used in the microcontroller; 40 cards used during setup to edit the text and characteristics of the cards used on the microcontroller; and 24 cards used to generate an unbounded amount of mana, draw all the remaining cards, set up the construction and remove all Bob’s cards.

With the correct draw, a player can take control of the game as early as the first turn, and set up the construction. Getting that correct draw is much less likely than with a 60-card deck, but this is a theoretical result anyway; the difference between a one in a million chance and a one in several trillion is not particularly relevant.

There are minimal constraints on Bob’s deck (one card to serve as the flag must have no abilities that function in the graveyard or hand), which will easily be satisfied by any normal deck. So it is perfectly possible for a hapless player to sit down expecting a tournament *Magic* game, have the opponent take over and set up the Microcontroller, and find that they can only win the game by winning (say) a game of chess instead.

6.3 Computational implications

The previous construction in [1] was Turing complete, so this does not increase the amount of computation possible inside *Magic*. However, the addition of input commands during

program execution adds a lot to the programs that can be usefully written, in terms of ability to simulate multi-player games involving choices – see e.g. sample program 4 which implements Nim. The language is clearly powerful enough to similarly write programs for chess, checkers, go, or any similar two-player perfect knowledge game.

A common joke upon the publication of [1] was “Now we can write *Magic Online* [a digital implementation of *Magic*] in *Magic*”. With the Turing machine-based construction, all players would have had to pre-register all their moves before computation started. By contrast, if a digital card game were implemented using the construction in this paper, players could choose their moves during gameplay in response to the moves made by their opponent.

Similarly, this result shows that the complexity of *Magic* includes any game or algorithm which involves a finite (but potentially unbounded) sequence of choices, including responses to another player’s choices. It is of course still not an especially practical environment to perform any real computation.

We include sample program 3, which searches for a counterexample to the Collatz conjecture, as a concrete demonstration of the possibility brought up in discussions of [1]. If Alice should set up the microcontroller and start this program, the game is a victory for Bob as soon as the program finds a cycle of numbers that is a counterexample to the Collatz conjecture. If (as is widely suspected) no such counterexample exists [9], or if it instead finds a sequence that goes on forever without repeating, the game is a draw by infinite loop. This provides an explicit *Magic* game state where all player choices have been removed but the end result of the game is unknown to current mathematics.

6.4 Further research

It is clear that *Magic* is as computationally complex as it’s possible for a perfect knowledge game to be. But not all two-player games are perfect knowledge, and *Magic* contains many cards and mechanics that use hidden information. Our construction doesn’t use any of these, but it’s possible future constructions could. This would allow embedding a wider variety of games into *Magic*, such as two-player games where both players choose their moves simultaneously.

It is also possible that there exist other tabletop games which support embedding this kind of construction. But any such game would need to be in the small subset of tabletop games which allow all of the following:

- An unlimited number of player actions rather than a fixed number of turns
- An unlimited number of at least one resource
- Some way to constrain the actions players can perform
- Enough flexibility in player choices to allow forcing one action to result in another

It may well be the case that *Magic* is the only widely played tabletop game meeting these criteria which has enough depth of rules and scope for player creativity to allow this kind of construction. If that is the case, we are very grateful to Wizards of the Coast for providing such a versatile set of building blocks for us to play with.

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Appendix A Sample Programs

Readers are encouraged to explore the functionality of these sample programs firsthand by executing them within the provided simulator interface [6] where they are preset options.

Sample Program 1 10 Cubed

| Symbols | Instruction | Comments |
|---------|--------------------|------------------------|
| 10 0 10 | NumBuild 10 | Initialise r_0 to 10 |
| 5 1 0 | Set $r_1=r_0$ | r_1 is the output |
| 6 0 1 | Multiply $r_1 r_0$ | Now r_1 is 100 |
| 6 0 1 | Multiply $r_1 r_0$ | Now r_1 is 1000 |
| 4 0 0 | HaltD | We're done. |

The complete program is: 10 0 10 5 1 0 6 0 1 6 0 1 4 0 0 – or in cards: Snow-Covered Forest, Plains, Snow-Covered Forest, Wastes, Island, Plains, Snow-Covered Plains, Plains, Island, Snow-Covered Plains, Plains, Island, Forest, Plains, Plains.

After 5 of Alice's turns and 4 of Bob's, the game will end in a draw. Register r_1 will have 1000 +1/+1 counters on it, the result of the calculation.

Sample Program 2 Prime Factors

| Symbols | Instruction | Comments |
|---|-------------------|---|
| <i>Register usage: r_0: constant 2. r_1: input number remaining to be factorised. r_2: copy of r_1 for divisions. r_3: current divisor. r_4: how many factors found so far.</i> | | |
| 2 1 0 | AInput 1 | Read the input number into r_1 |
| 4 1 3 | Add1 r_3 | Initialise divisor to 1 |
| 10 0 2 | NumBuild 2 | Initialise r_0 to constant 2 |
| <i>Main loop: test the next number</i> | | |
| 4 1 3 | Add1 r_3 | Increment the divisor we're testing |
| 5 2 1 | Set $r_2 r_1$ | Prepare to test r_1 |
| 7 2 3 | DivCeil $r_2 r_3$ | Divide and check remainder |
| 3 5 4 | JumpBwdF 4 | If flag, r_3 is not a factor |
| <i>Found a factor: store it and the quotient</i> | | |
| 8 3 4 | Store $r_3 r_4$ | It is. Save r_3 to a new memory slot, |
| 4 1 4 | Add1 r_4 | and increment number of factors found |
| 5 1 2 | Set $r_1 r_2$ | Remember the new divided total |
| 11 0 2 | FLess $r_2 r_0$ | Is r_2 now 1? |
| 3 3 8 | JumpBwdNF 8 | If not, continue. There might be another factor of r_3 so check it again. |
| 4 0 0 | HaltD | If so, halt |

After execution finishes, there will be one memory entry for each prime factor in the input number. For example, if Alice chooses 120, the program finishes after 57 turn cycles, and memory consists of {2, 2, 2, 3, 5}.

■ **Sample Program 3** Collatz ($3n + 1$)

| Symbols | Instruction | Comments |
|--|----------------------------|--|
| <i>Register usage: r_0: built numbers. r_1, r_2, r_3: constants 1, 2, 3. r_4: source of the current chain. r_6: temp read memory. r_7: current number being checked. r_8: either half r_7 or $3r_7 + 1$.</i> | | |
| 4 1 1 | Add1 r_1 | Initialise r_1 to 1 |
| 10 0 2 | NumBuild 2 | Create constant 2 |
| 5 2 0 | Set r_2 r_0 | Store constant 2 in r_2 |
| 10 0 3 | NumBuild 3 | Create constant 3 |
| 5 3 0 | Set r_3 r_0 | Store constant 3 in r_3 |
| 10 4 2 | NumBuild $12 \times 4 + 2$ | Start searching at 50 |
| 5 4 0 | Set r_4 r_0 | Initialise current search root |
| 5 7 4 | Set r_7 r_4 | Start checking at current search root |
| <i>Label 0: we have a new r_7 to investigate</i> | | |
| 11 2 7 | FLess r_7 r_2 | Is $r_7 = 1$? |
| 10 1 4 | NumBuild $12 \times 1 + 4$ | Create longjump distance 16 |
| 3 4 0 | JumpFwdF r_0 | If so, jump to label 3 |
| 9 6 7 | Load r_6 r_7 | Load memory r_7 into r_6 |
| 11 6 6 | FIsZero r_6 | Is this a new number? |
| 3 4 3 | JumpFwdF 3 | If so, go to label 1 |
| 11 2 6 | FLess r_6 r_2 | Is this a number that we know gets to 1? |
| 3 4 11 | JumpFwdNF 1 | If so, jump to label 3 |
| 4 0 2 | HaltB | If not, we've found a loop and disproved the Collatz conjecture! |
| <i>Label 1: r_7 is a number we've not seen before</i> | | |
| 5 8 7 | Set r_8 r_7 | Prepare to halve r_8 |
| 4 2 8 | Halve r_8 | Halve r_8 . Did that leave remainder? |
| 3 2 3 | JumpFwdNF 3 | If not, r_8 is what we want at Label 2 |
| 5 8 7 | Set r_8 r_7 | Set r_8 to r_7 ... |
| 6 3 8 | Mult r_8 r_3 | ... $\times 3$... |
| 4 1 8 | Add1 r_8 | ...+1. |
| <i>Label 2: r_8 is the next number in the sequence</i> | | |
| 8 8 7 | Store r_7 r_8 | Store r_7 in memory r_8 |
| 5 7 8 | Set r_7 r_8 | Now investigate r_8 |
| 10 1 7 | NumBuild $12 \times 1 + 7$ | Create longjump distance 19 |
| 3 1 0 | JumpBwd r_0 | Go back to label 0 |
| <i>Label 3: A number r_4 got down to 1. Label the chain with 1s.</i> | | |
| 5 7 4 | Set r_7 r_4 | Restart at r_4 |
| 9 6 7 | Load r_6 r_7 | Load memory r_7 into r_6 |
| 11 2 6 | FLess r_6 r_2 | Is $r_6 = 1$? |
| 3 4 3 | JumpFwdF 3 | If so, skip to end of the loop |
| 8 1 7 | Store r_7 r_1 | Save 1 into r_7 |
| 5 7 6 | Set r_7 r_6 | Set r_7 to the number we read |
| 3 1 6 | JumpBwd 6 | Go back 6 |
| 4 1 4 | Add1 r_4 | We're done with r_4 's chain. Next number! |
| 3 0 7 | JumpFwd 7 | Loop around to just before label 0 |

18:18 A Programming Language Embedded in *Magic: The Gathering*

■ Sample Program 4 Nim

| Symbols | Instruction | Comments |
|---|--------------------|--|
| <i>Register usage: r_0 built numbers. r_1 number of piles. r_2 loop index. r_4 current player. r_5 chosen pile. r_6 pile value. r_7 number to remove. r_8 temp comparison. r_9 constant long jump distance 27. r_{11} constant 1.</i> | | |
| <i>Initialise variables</i> | | |
| 10 0 1 | NumBuild 0 1 | $r_0=1$ |
| 5 11 0 | Set r_{11} r_0 | $r_{11}=1$ |
| 10 2 3 | NumBuild 2 3 | $r_0=2 * 12 + 3=27$ |
| 5 9 0 | Set r_9 r_0 | $r_9=27$ |
| <i>Initialise: Input number of piles</i> | | |
| 2 1 0 | AInput r_1 | Ask A for number of piles N |
| <i>Loop asking Alice for initial values of piles</i> | | |
| 5 2 11 | Set r_2 r_{11} | Initialise r_2 to 1 |
| <i>Label 0: input one initial pile size</i> | | |
| 2 6 0 | AInput r_6 | Ask A for count of pile r_2 |
| 8 6 2 | Store r_2 r_6 | Save r_6 into memory r_2 |
| 4 1 2 | Add1 r_2 | Increment r_2 |
| 11 2 1 | FLess r_1 r_2 | If r_2 is still $\leq r_1$, |
| 3 3 5 | JumpBwdNF 6 | jump back to label 0 |
| 5 4 4 | Zero r_4 | Set current player to A |
| <i>Label 1: Main game loop</i> | | |
| <i>Toggle current player $r_4=1-r_4$</i> | | |
| 1 4 4 | Sub1 r_4 | $r_4 -= 1$ |
| 3 2 1 | JumpFwdNF 1 | Jump forward 1 if subtraction didn't go negative |
| 5 4 11 | Set r_4 r_{11} | Set r_4 to 1 |
| <i>Label 2: Ask current player which pile to take from</i> | | |
| 11 4 4 | FIsZero 4 | Is $r_4 = 0$? |
| 3 2 2 | JumpFwdNF 2 | Jump forward 2 if not |
| 2 5 0 | AInput r_5 | If r_4 is 0, ask Alice a pile number |
| 3 0 1 | JumpFwd1 | Only if r_4 is >0 , |
| 2 5 1 | BInput r_5 | Ask Bob a pile number |
| <i>Ensure that pile's not empty</i> | | |
| 9 6 5 | Load r_5 r_6 | Load memory r_5 into r_6 |
| 1 6 6 | Sub1 r_6 | Is r_6 0? |
| 3 5 8 | JumpBwdF 8 | If so, jump back to label 2 (-8) |
| <i>Label 3: Ask current player how much to remove</i> | | |
| 9 6 5 | Load r_5 r_6 | Load memory r_5 into r_6 |
| 11 4 4 | FIsZero 4 | Is $r_4 = 0$? |
| 3 2 2 | JumpFwdNF 2 | Jump forward 2 if not |
| 2 7 0 | AInput r_7 | If r_4 is 0, ask Alice a number of sticks |
| 3 0 1 | Jump forward 1 | Only if r_4 is >0 , |
| 2 7 1 | BInput r_7 | Ask Bob a number of sticks |
| <i>Check they didn't say 0</i> | | |
| 11 7 7 | FIsZero r_7 | Is $r_7 = 0$? |

| Symbols | Instruction | Comments |
|---|----------------------|--|
| 3 5 8 | JumpBwdF 8 | If so, jump back to L3 (-8) |
| <i>Try to remove r_7 from the pile</i> | | |
| 1 7 6 | Subtract r_7 r_6 | Try to remove r_7 from pile r_6 |
| 3 5 10 | JumpBwdF 4 | If that failed, ask again at L3 |
| 8 6 5 | Store r_5 r_6 | Save new pile value in memory r_5 |
| <i>Player has successfully taken a turn.</i> | | |
| <i>Ensure we have at least one pile remaining</i> | | |
| 5 2 11 | Set r_2 r_{11} | Initialise $r_2=1$ |
| 5 0 9 | Set r_0 r_9 | Store long-jump distance in r_0 |
| <i>Label 4: check if pile r_2 still exists</i> | | |
| 9 6 2 | Load r_6 r_2 | Load memory r_2 into r_6 |
| 11 6 6 | FIsZero r_6 | Is this pile's value 0? |
| 3 3 0 | JumpBwdNF 0 | If not, jump back to L1 (r_0 long jump) |
| 4 1 2 | Add1 r_2 | OK, keep looking for a nonempty pile |
| 5 8 1 | Set r_8 r_1 | Has r_2 reached the number of piles? |
| 1 2 8 | Subtract r_2 r_8 | See if $r_2 >$ number of piles |
| 3 3 7 | JumpBwdNF 7 | If not, jump back to L4 |
| <i>Game over. Last one to play wins.</i> | | |
| 11 4 4 | FIsZero r_4 | Is r_4 0? |
| 3 4 1 | JumpFwdF 1 | If so, jump forward 1 |
| 4 0 2 | HaltB | Bob wins! |
| 4 0 1 | HaltA | Alice wins! |

Appendix B Implementation of the Programming Instructions

The detailed explanations provided below are also accessible via our supplementary webpage [4], where the content is presented with enhanced readability through dynamic tooltips which provide additional information.

B.1 0 Y Z – Add r_Y r_Z – Set r_Y to $r_Y + r_Z$.

- Bob has **Halana and Alena, Partners**, with added creature types X_0 Elf tr_Y , and with P/T set to 0/2.

In Bob's combat phase, the **Halana and Alena, Partners** triggers. It targets r_Y , and its power is boosted by r_Z to the value of r_Z , thus adding that value to r_Y .

B.2 5 Y Z ($Y \neq Z$) – Set r_Y r_Z – Set r_Y to the value of r_Z .

- Bob has **Halana and Alena, Partners**, with added creature types X_5 Elf tr_Y , and with P/T set to 0/2.
- Alice has an **Archpriest of Iona** with added creature types X_5 Y_A tr_Y .
- Alice has a green and black 3/3 **Shape Stealer** with added creature types X_5 Y_A , given flying and with a -1/-0 counter on it.
- She also has a 0/2 Warrior token, a 0/2 Rogue token and a 0/2 Wizard token.

This instruction is implemented similarly to the previous one, except that we need to erase the previous value of r_Y before adding the value of r_Z .

To do this, in Alice's third combat phase, since Alice has a full party, **Archpriest of Iona** triggers and gives r_Y flying and +1/+1. The **Shape Stealer** attacks and is blocked by the register, taking the register's power as its base power, and the extra 1 is subtracted off, so that its power equals the register's value. It is given wither because it is black, so the damage is dealt as -1/-1 counters, cancelling out all the +1/+1 counters on the register.

In Bob's combat phase, **Halana and Alena, Partners** adds r_Z +1/+1 counters to r_Y . (Note that despite having reach, this couldn't block the **Shape Stealer** because of the tr_Y types, as protection also restricts blocking.)

B.3 5 $Y Y$ – Zero r_Y – Set r_Y to zero.

When Y and Z are the same, something different happens. Because the register's value is erased first, it isn't there to get added in Bob's combat phase, leaving it with no +1/+1 counters.

B.4 1 $Y Z$ ($Y \neq Z$) – SubCond $r_Z r_Y$ – If $r_Z \geq r_Y$, set r_Z to $r_Z - r_Y$ and set the flag to 0. Otherwise, set the flag to 1.

- Alice has **Necrogen Mists** given type X_1 .
- Bob has a 1/1 **Halfdane** with added copiable types $X_1 tr_Y$, given islandwalk, to which is attached a **Tanuki Transplanter** controlled by Alice.
- Bob has a 0/1 green and black **Furtive Homunculus**, with added copiable types X_1 Elf Dragon tR, and given flying and “Whenever this creature deals combat damage to a player, draw a card.”
- Alice has a 0/1 **Omnath, Locus of Mana**, with added creature types X_1 , given reach and indestructible.
- Bob has a **Wrathful Red Dragon**, with added creature types X_1 and tr_Z , with flying removed.
- Bob has **Belligerent Brontodon** with added creature type X_1 .

In Bob's upkeep, **Necrogen Mists** makes Bob discard the flag card if it's in hand, and **Halfdane** triggers and targets r_Y , taking on its power and toughness.

In Bob's combat phase, **Halfdane** and **Furtive Homunculus** attack, and **Tanuki Transplanter** triggers, giving Alice r_Y 's value in green mana, which makes that the power of **Omnath, Locus of Mana**.

The **Furtive Homunculus**, with r_Z power and skulk, is blocked by **Omnath, Locus of Mana** if $r_Y \leq r_Z$. If it blocks, it deals r_Y damage to the **Furtive Homunculus**, triggering **Wrathful Red Dragon** to make the **Furtive Homunculus** deal r_Y damage with wither to r_Z , reducing its value by r_Y ; the combat damage from the **Furtive Homunculus** to **Omnath, Locus of Mana** is prevented because of the former's tR type. If it doesn't block, **Belligerent Brontodon** ensures that the **Furtive Homunculus** deals at least 1 damage to Alice even if $r_Z=0$, triggering the added ability to make Bob draw a card and set the flag to 1.

The green mana disappears at the end of Alice's untap step in the next turn, when **Omnath, Locus of Mana** is no longer active.

B.5 1 Z Z – Sub1Cond r_Z – If $r_Z \geq 1$, set r_Z to $r_Z - 1$ and set the flag to 0. Otherwise, set the flag to 1.

- Bob has a **Riders of Gavony** with type set to X_1 and with text changed to give Rabbit creatures protection from Yetis at timestamp ①.

This is a special case of the previous instruction. This **Riders of Gavony** gives protection from Yetis to all the registers, but only r_Z gets to keep it after **Blood Sun** removes abilities from lands. If $Y = Z$, this leaves the **Halfdane** from the previous section with no legal targets for its ability, and its power remains 1, making 1 the value to be subtracted.

B.6 6 Y Z – Mult $r_Z r_Y$ – Set r_Z to $r_Y \times r_Z$.

- Alice has an **Archpriest of Iona** with added creature types $X_6 Y_A tr_Y$.
- Alice has a 2/2 indestructible **Smog Elemental** with added creature types $X_6 Y_A$.
- Alice has a **Wrathful Red Dragon** with added creature types $X_6 tU$, with flying removed and edited to trigger on Elementals.
- Alice has another **Wrathful Red Dragon**, this one with added creature types $X_6 tr_Z$, also with flying removed.
- Bob has a blue indestructible **Hornet Nest** with type X_6 and edited to create Elf tokens.
- Bob has a 1/1 **Phantom Steed** given islandwalk, edited to add type Elf to the token, and exiling a **War Elemental**.
- Alice has a 0/1 indestructible blue, black and red **Tocatli Honor Guard** with added creature types X_6 Dragon.
- Alice has a **Ward Sliver** naming green, edited to give Elementals protection from green, and with added creature type X_6 .
- Bob has a green and blue **Resourceful Defense** given types $X_6 tr_Z$.

In Alice's third combat phase, **Archpriest of Iona** triggers and gives r_Y flying and +1/+1; the +1/+1 gets cancelled out by the -1/-1 from **Smog Elemental**.

The **Smog Elemental** attacks and gets blocked by r_Y . It receives the value of r_Y in damage. This triggers the first **Wrathful Red Dragon** to make it deal the same amount of damage to the **Hornet Nest**, which produces r_Y 1/1 green Elf creature tokens with flying and deathtouch. Affected by the **Smog Elemental** and r_Z , their power and toughness are equal to r_Z 's value. (If that's 0, they die immediately.)

In Bob's combat phase, those Elf tokens attack, along with the **Phantom Steed**, which creates an attacking Elf token copy of **War Elemental**; **Tocatli Honor Guard** suppresses its enters-the-battlefield trigger. That token is given protection from green by the **Ward Sliver**, so the **Tocatli Honor Guard** is the only creature that can block it, and does so; the protection also stops it from being targeted later.

In the combat damage step, the **War Elemental** token deals $r_Z + 1$ damage to the **Tocatli Honor Guard**, triggering the second **Wrathful Red Dragon** to make it deal the same amount of damage with wither to r_Z , which leaves it with one -1/-1 counter; the Elf tokens are reduced to 0 toughness and die. The **Phantom Steed** and the Elf tokens deal their damage to Alice, triggering the **War Elemental** to get $r_Y \times r_Z + 1$ +1/+1 counters.

At the end of combat, the **War Elemental** token is sacrificed, triggering **Resourceful Defense**, which targets r_Z (being blue stops it from targeting Bob's **Sterling Grove**) to put $r_Y \times r_Z + 1$ +1/+1 counters on it; one gets cancelled out and its new value is $r_Y \times r_Z$.

B.7 $7 Y Z - \text{DivCeil } r_Y r_Z - \text{Set } r_Y \text{ to } \lceil r_Y/r_Z \rceil$. If the division was exact, set the flag to 0, otherwise set it to 1. If $r_Z = 0$ or $Y = Z$, this is undefined behaviour.

- Alice has a **Sporemound** with added creature types $X_7 Y_A$ edited to create a blue Volver token.
- Alice has **Engineered Plague** given type X_7 affecting Volvers.
- Alice has a **Chief of the Scale** with added creature types $X_7 Y_A$ edited to affect Volvers.
- Alice has an **Archpriest of Iona** with added creature types $X_7 Y_A tr_Y$.
- Alice has a 3/3 green and black **Shape Stealer** with added creature types $X_7 Y_A$, given flying and indestructible and with a -1/-0 counter on it.
- Bob has a **Questing Beast** with added creature type X_7 .
- Bob has a **Wrathful Red Dragon** with added creature types $X_7 tU$, edited to trigger on Rabbits, with flying removed.
- Alice has a **Vigor** with added creature types $X_7 Y_A$.
- Alice has a **Necrogen Mists** given type X_7 .
- Bob has a nontoken **Progenitor Mimic** copying **Polyraptor** with added copiable types Elf Sand, copiously made a 0/0 creature, with type noncopiously set to X_7 , and with a +1/+1 counter on it.
- Bob has a **Gruul Ragebeast** with added creature types $X_7 tU$.
- Bob has a **Sliver Hivelord** with type noncopiously set to X_7 edited to affect Sands.
- Bob has a -1/2 **Halana and Alena, Partners** with added creature types $X_7 tr_Y$, to which is attached Bob's **Raised by Wolves** edited to count Sands.
- Bob has **Aegar, the Freezing Flame** with type noncopiously set to X_7 edited to trigger on Sands.

In Alice's third combat phase, **Archpriest of Iona** triggers and targets r_Y . Then **Shape Stealer** attacks, along with the regular attackers. The new program permanent entering the battlefield triggers **Sporemound**, which creates a 1/1 blue Volver. It has -1/-1 from **Engineered Plague** and +0/+1 from Alice's **Chief of the Scale**.

As we've seen before, r_Y blocks the **Shape Stealer**, and the **Shape Stealer** takes on r_Y 's power and toughness. **Shape Stealer** deals r_Y 's value in damage with wither to r_Y . Bob's **Wrathful Red Dragon** triggers and targets the blue Volver token. Alice's **Vigor** turns this damage into +1/+1 counters so the blue Volver has $r_Y +1/+1$ counters on it, and r_Y is reset to 0. (r_Y 's damage to the **Shape Stealer** can't be prevented by **Vigor** because of Bob's **Questing Beast**, but **Shape Stealer** is indestructible. **Wrathful Red Dragon**'s damage is still affected by **Vigor** because **Questing Beast** only affects combat damage.)

In Bob's turn, Alice's **Vigor** and **Chief of the Scale** become inactive. In his upkeep, both his **Progenitor Mimic** and Alice's **Necrogen Mists** trigger. APNAP order means first, **Necrogen Mists** makes him discard his card if he has one.

Then **Progenitor Mimic** creates him a **Polyraptor** token with power and toughness each r_Z . **Gruul Ragebeast** triggers and makes the **Polyraptor** token fight Alice's Volver, dealing r_Z damage to it. **Sliver Hivelord** means the **Polyraptor** is indestructible, but it still takes damage from the Volver, so its ability triggers. This creates a new indestructible **Polyraptor** which again triggers **Gruul Ragebeast**, and this continues until the Volver has taken enough damage to die. At that point there are no legal targets for **Gruul Ragebeast**'s ability so the loop stops. We will always have $\lceil r_Y/r_Z \rceil + 1$ **Polyraptors** created this way. If the last instance of damage was excess damage – i.e. there's a remainder from the division – then **Aegar, the Freezing Flame** will make Bob draw a card, setting the flag.

At the beginning of combat on Bob's turn, **Halana and Alena, Partners** triggers targeting r_Y and gives it a number of +1/+1 counters equal to one less than the number of Sand tokens, which works out as $\lceil r_Y/r_Z \rceil$.

Once Bob's turn ends, all the registers will be inactive, so Bob's Elf Sand tokens will no longer have anything increasing their toughness, and they automatically die.

B.8 8 Y Z – Store r_Z r_Y – Store the value of r_Y at memory address r_Z .

- Bob has a **Halfdane** with added creature types X_8 *tr_Y*, given *islandwalk* and *trample*.
- Alice has a **Godhead of Awe** with added creature type X_8 , timestamp ① with *flying removed*.
- Bob has a **Wandering Wolf** with added creature types X_8 *Elf*, given *flying* and *indestructible*.
- Bob has **Behind the Scenes** given type X_8 .
- Bob has **Spinneret Sliver** with type set to X_8 but edited to affect *Mouse* creatures.
- Bob has **Quartzwood Crasher** with added creature type X_8 edited to create *Mouse* tokens.
- Bob has a 2/2 **Arwen, Weaver of Hope** with added creature types X_8 *Elf*.
- Alice has a green and black **Aether Flash** given type X_8 , to which is attached a **Charisma** controlled by Alice.

In Bob's upkeep, **Halfdane** targets r_Y and takes on its power and toughness. (Note that the **Godhead of Awe** has an early timestamp of ①, so it doesn't affect the final power and toughness of the registers, which have their base power and toughness set at ① and possibly later.)

In Bob's combat phase, the **Halfdane** and **Wandering Wolf** attack. The **Wandering Wolf** has $r_Z + 1$ power, and its own ability plus skulk from **Behind the Scenes** together mean that it can be blocked only by creatures with exactly the same power; thus, the existing memory token for address r_Z blocks if it exists (given reach by **Spinneret Sliver**).

Combat damage is dealt; the blocking memory token (if present) is dealt exactly enough damage to be destroyed. **Halfdane** deals r_Y damage to Alice and triggers **Quartzwood Crasher** if that's nonzero, which produces a *Mouse* creature token with that power and toughness. **Arwen, Weaver of Hope** adds $r_Z + 2$ +1/+1 counters to it. **Aether Flash** triggers and deals 2 damage with *wither* to the token, cancelling out the extra two +1/+1 counters and triggering **Charisma** to give Alice control of it.

B.9 9 Y Z – Load r_Y r_Z – Load the value at memory address r_Z into r_Y .

- Bob has **Archpriest of Iona** with added creature types X_9 *tr_Y*.
- Bob has a **Dream Fighter** with added creature types X_9 *Sliver tR*, given *flying*.
- Bob has a 2/2 **Wandering Wolf** with added creature types X_9 *Elf Dragon*, given *flying* and *indestructible*, to which is attached a **Skeleton Key** controlled by Bob.
- Alice has a **Godhead of Awe** with added creature types X_9 *Sliver*, timestamp ①, with *flying removed* but granted *reach* instead.
- Alice has a green and black **Shape Stealer** with added creature types X_9 *tR* and *reach*, with a -1/-0 counter on it.
- Bob has **Spinneret Sliver** with type set to X_9 but edited to affect *Mouse* creatures.

- Bob has **Serpent of Yawning Depths** with type X_9 but edited to affect creatures with type Rabbit or tR .
- Bob has **Field Marshal** with type X_9 but edited to affect Mouse creatures.
- Alice has **Questing Beast** with type X_9 .
- Bob has a blue $1/1$ **Vigor** with added creature types X_9 Elf given indestructible.
- Bob has a **Wrathful Red Dragon** with types X_9 tU and flying removed.
- Alice has **Toralf, God of Fury** with added creature types X_9 tr_Y .
- Alice has **Gideon's Intervention** with type X_9 naming Wandering Wolf.

In Bob's combat phase, the **Archpriest of Iona** gives $+1/+1$ and flying to r_Y . That register attacks, along with the **Dream Fighter** and the **Wandering Wolf**. (The **Dream Fighter** has been given the type tR to stop it from blocking the creatures of the conditional mechanism in Alice's turn.)

In the declare blockers step, the **Godhead of Awe** blocks the **Dream Fighter**, because those are given shadow by the **Shadow Sliver**. The **Shape Stealer** blocks r_Y ; it can only block red creatures because of its tR type, and it is the only creature that can block r_Y because of the **Serpent of Yawning Depths**. Finally, the **Wandering Wolf**, because of its own ability and skulk from **Skeleton Key**, can only be blocked by creatures with exactly the same power. Its power is $r_Z + 2$, and each memory token (given reach by **Spinneret Sliver**, and affected by **Godhead of Awe** and **Field Marshal**) has power equal to 2 plus its address, so the one with address r_Z blocks if it exists.

This blocking triggers two abilities: the **Shape Stealer** takes on r_Y 's power as its base power, ending up with r_Y 's value as its power, and the **Dream Fighter** phases out itself and the **Godhead of Awe**, which, most importantly, uncovers the original base power and toughness of the memory tokens.

The blocking memory token (if it exists), given first strike and $+1/+1$ by **Field Marshal**, deals its damage first, dealing $\text{address} + \text{value} + 1$ damage to the **Wandering Wolf**; **Questing Beast** stops **Vigor** from preventing this damage. **Wrathful Red Dragon** triggers and targets **Vigor**, making **Wandering Wolf** deal the same amount of damage to it (it does not prevent damage to itself). The **Vigor**'s toughness is $r_Z + 1$, so if the value is greater than zero this triggers **Toralf, God of Fury** to deal the value in damage to r_Y ; this damage is not combat damage so it is prevented by **Vigor**, adding the value in $+1/+1$ counters to r_Y .

In the regular combat damage step, the **Shape Stealer** deals its damage with wither to r_Y (again, **Questing Beast** stops **Vigor** from preventing it), subtracting off its original value and leaving just the memory value. **Gideon's Intervention** prevents the damage from **Wandering Wolf**, stopping it from killing the memory token or triggering the **Skeleton Key**'s second ability.

B.10 2 Y 1 – BInput r_Y – Set r_Y to a nonnegative integer of Bob's choice.

- Alice has **Archpriest of Iona** with added creature types X_2 Y_A tr_Y .
- Alice has a green and black $3/3$ **Shape Stealer** with added creature types X_2 Y_A , given flying and with a $-1/-0$ counter on it.
- Alice has a $0/4$ **Questing Beast** with added creature types X_2 Dragon given reach and indestructible.
- Alice has a blue **Wrathful Red Dragon** with added creature types X_2 tr_Y with flying removed.

- Bob has **Vigor** with added creature type X_2 .
- Bob has a nontoken **Progenitor Mimic** copying **Volcano Hellion** with added copiable types $X_2 Z_1 tU$, copiously made a 0/0 artifact creature, with a +1/+1 counter on it.

We start by clearing the value of r_Y , in a way we've seen before, by giving it flying and having it block a **Shape Stealer**; **Questing Beast** stops **Vigor** from preventing the damage. This part is shared between multiple instructions that start with card 2.

In Bob's upkeep, the **Volcano Hellion** produces a copy of itself, which dies immediately and targets the **Wrathful Red Dragon** with its ability. Bob gets to choose how much damage it deals to him and to the **Wrathful Red Dragon**, which triggers (if the amount is nonzero) and targets r_Y ; that damage is turned into +1/+1 counters by **Vigor**.

(The unpreventability of **Volcano Hellion's** damage is the reason for the indirection through the **Wrathful Red Dragon**. However, unpreventability doesn't stop **Worship** from modifying the *result* of the damage dealt to Bob.)

B.11 2 Y 0 – Aninput r_Y – Set r_Y to a nonnegative integer of Alice's choice.

- Alice has a **Volcano Hellion** with added copiable types $X_2 Z_0 tU$, copiously made a 0/0 artifact creature, with a +1/+1 counter on it, and noncopiously coloured red and with added creature type Dragon.
- Alice has **Artificer Class** at level 3 given types $X_2 Z_0 tR$.

This is similar to the previous instruction, but it's a little more complicated because Alice's upkeep is not usable. This time, **Artificer Class** is used to make the copy of **Volcano Hellion** in Alice's end step, and the rest proceeds similarly to before; the "isn't a Dragon" targeting criterion of **Wrathful Red Dragon** stops it from targeting this **Volcano Hellion**.

B.12 2 Y 2 – SetF r_Y – Set r_Y to the flag's value.

- Bob has **Syr Elenora, the Discerning** with added creature types $X_2 Z_2$ given flying. **Syr Elenora, the Discerning** attacks and gets blocked by the **Questing Beast** from B.10. If the flag is 1, it deals 1 damage and triggers the **Wrathful Red Dragon**, which adds 1 to r_Y as before.

B.13 2 Y 3 – SetNF r_Y – Set r_Y to the Boolean negation of the flag's value (1 if it's 0 and vice versa).

- Bob has a 1/1 **Slithering Shade** with added creature types $X_2 Z_3$. The **Slithering Shade** attacks if the flag is 0, making r_Y 1 as before.

B.14 2 Y 4 – Rand6 r_Y – Set r_Y to a uniformly random nonnegative integer less than 6.

- Bob has a -1/1 **Suntail Hawk** with added creature types $X_2 Z_4$, to which is attached a **Strength-Testing Hammer**.

The **Suntail Hawk** attacks, and the **Strength-Testing Hammer** triggers and increases its power to the required random number. **Vigor's** 6 power ensures that the "draw a card" effect never happens.

B.15 2 Y 5 – Rand20 r_Y – Set r_Y to a uniformly random nonnegative integer less than 20.

- Bob has an **Ancient Gold Dragon** with types set to $X_2 Z_5$ edited to create Sand tokens, given islandwalk and first strike.
- Bob has a $-1/1$ **Rat Colony** with types set to $X_2 Z_5$, edited to count Sand creatures, and given flying.
- Bob has a **Goblin Pyromancer** with types set to $X_2 Z_5$ edited to affect Sand creatures. The **Ancient Gold Dragon** and **Rat Colony** attack, and the **Rat Colony** is blocked by the **Questing Beast**. The **Ancient Gold Dragon** first deals its damage to Alice and randomly produces 1-20 Sand tokens, which increase the **Rat Colony**'s power to the required random number, and it then deals its damage, which goes into r_Y 's value as before. In Bob's end step, **Goblin Pyromancer** gets rid of the Sand tokens.

B.16 4 0 0 – HaltD – End the game in a draw.

- Bob has a **Celestial Convergence** given types $X_4 Y_0$ with 1 omen counter on it. **Celestial Convergence** triggers in Bob's upkeep, removes its last omen counter, and compares life totals. Both players' life totals are 1, so the game ends in a draw.

B.17 4 0 1 – HaltA – End the game with Alice winning.

- Alice has an **Alert Heedbonder** with added creature types $X_4 Y_0 Z_1$. **Alert Heedbonder** triggers in Alice's end step and Alice gains 1 life from it counting itself, and then Alice wins from the **Celestial Convergence** trigger.

B.18 4 0 2 – HaltB – End the game with Bob winning.

- Alice has a **Spiritual Sanctuary** given types $X_4 Y_0 Z_2$, edited to count Forests. **Spiritual Sanctuary** triggers in Bob's upkeep; APNAP order means that trigger goes on top of the **Celestial Convergence** trigger, and resolves first. Bob does control a Forest, so Bob gains 1 life, and wins from the **Celestial Convergence** trigger.

B.19 4 1 Z – Add1 r_Z – Set r_Z to $r_Z + 1$.

- Bob controls a $1/1$ **Halana and Alena, Partners** with added creature types $X_4 Y_1 tr_Z$. **Halana and Alena, Partners** targets r_Z and adds one $+1/+1$ counter to it.

B.20 4 2 Z – Halve r_Z – Set r_Z to half the value of r_Z , rounding down. Set the flag to the remainder from the division.

- Alice has a $1/1$ **Abyssal Specter** with added creature types $X_4 Y_2$ given first strike.
- Alice has a $2/2$ green and black **Catacomb Dragon** with added creature types $X_4 Y_1 tr$.
- Bob has a $0/1$ blue **Taii Wakeen, Perfect Shot** with added creature types $X_4 Y_2 Elf$ given reach and indestructible.
- Alice has a **Wrathful Red Dragon** with added creature types $X_4 Y_2 tr_Z$ with flying removed.
- Bob has a **Wrathful Red Dragon** with added creature types $X_4 Y_2 tU$ with flying removed, edited to trigger on Rabbits.

In Alice's third combat phase, the **Catacomb Dragon** and **Abyssal Specter** attack, in addition to the conditional mechanism's creatures. The **Catacomb Dragon** is blocked by **Taii Wakeen, Perfect Shot**.

By APNAP order, the trigger from Bob's **Dream Fighter** in the conditional mechanism (subsection 4.4) is put on top of the **Catacomb Dragon**'s trigger on the stack, and thus resolves first; therefore, by the time the **Catacomb Dragon**'s trigger resolves, r_Z is active and is adding its value to the power and toughness of **Taii Wakeen, Perfect Shot**.

Let's say the value is V . The **Catacomb Dragon** subtracts $\lfloor V/2 \rfloor$ from **Taii Wakeen, Perfect Shot**'s power, leaving $V - \lfloor V/2 \rfloor = \lceil V/2 \rceil$ power.

In the first combat damage step, the **Abyssal Specter** deals its damage to Bob, and triggers to make Bob discard a card, setting the flag to 0.

In the second combat damage step, the **Catacomb Dragon**'s damage is prevented because of its tR type, and **Taii Wakeen, Perfect Shot** deals $\lceil V/2 \rceil$ damage. That damage triggers Alice's **Wrathful Red Dragon**, which targets r_Z and makes **Catacomb Dragon** deal $\lceil V/2 \rceil$ damage to it with **wither**, reducing its value to $V - \lceil V/2 \rceil = \lfloor V/2 \rfloor$.

That damage, in turn, triggers Bob's **Wrathful Red Dragon**, which targets **Taii Wakeen, Perfect Shot** and makes r_Y deal $\lceil V/2 \rceil$ damage to it. This damage is checked against its current toughness of $\lfloor V/2 \rfloor + 1$ (but does not destroy it because it has indestructible). If V is odd, the values are equal, triggering **Taii Wakeen, Perfect Shot** to make Bob draw a card, which sets the flag to 1.

B.21 10 $Y Z$ – NumBuild ($12Y + Z$) – Set r_0 to $12Y + Z$, except if the last instruction that was executed was also a NumBuild instruction, in which case multiply r_0 by 144 and add $12Y + Z$ to it.

- Alice has an **Excruciator** with added creature types $X_{10} X_A$, given flying, indestructible and vigilance.
- Alice has a $2/2$ blue **Dream Fighter** with added creature type X_{10} , given flying and protection from red.
- Bob has **Vigor** with added creature types $X_{10} tU$ given reach and islandwalk.
- Bob has **Khorvath Brightflame** with added creature type X_{10} , with flying removed, and edited to affect Rhinos.
- Bob also has **Sylvia Brightspear** with added creature type X_{10} , edited to affect Rhinos.
- Alice has a $0/1$ green and black **Mangara's Equity** with added creature types X_{10} Nomad, given reach, naming red, and edited to trigger on damage to green creatures.
- Alice has a $0/2$ **Darksteel Myr** with added creature types $X_{10} Y_0$.
- Alice has a $12/2$ **Darksteel Myr** with added creature types $X_{10} Y_1$.
- Alice has a $24/2$ **Darksteel Myr** with added creature types $X_{10} Y_2$.
- ⋮
- Alice has a $132/2$ **Darksteel Myr** with added creature types $X_{10} Y_{11}$.
- Bob has **Ojutai, Soul of Winter** with added creature types $X_{10} tU$, with flying removed, edited to trigger on Rhinos.
- Bob has **Angrath's Marauders** with added creature types $X_{10} Z_A$.
- Alice has a **Kangee, Aerie Keeper** with flying removed, with added creature type Z_1 , edited to affect Nomads, and with one feather counter.
- Alice has a **Kangee, Aerie Keeper** with flying removed, with added creature type Z_2 , edited to affect Nomads, and with two feather counters.

- Alice has a **Kangee, Aerie Keeper** with flying removed, with added creature type Z_3 , edited to affect **Nomads**, and with three feather counters.
⋮
- Alice has a **Kangee, Aerie Keeper** with flying removed, with added creature type Z_{11} , edited to affect **Nomads**, and with eleven feather counters.
- Bob has a **Angrath's Marauders** with added creature types $X_{10} Z_A$.

As its name suggests, this instruction can be used repeatedly to build any nonnegative integer value in r_0 , two base-12 digits at a time.

If X_{10} was read in Alice's first combat phase, then in her second combat phase, **Excruciator** and **Dream Fighter** attack. **Vigor** blocks **Dream Fighter** (its tU type stops it from blocking **Excruciator**), which triggers to phase both out. r_0 , given flying (and haste) by **Khorvath Brightflame** and double strike by **Sylvia Brightspear**, blocks the **Excruciator** (**Dream Fighter**'s protection from red stops r_0 from blocking it). In the first combat damage step, r_0 deals its value in damage, triggering **Mangara's Equity** to deal the same amount of damage back to it with **withr**, resetting its value to 0. (r_0 then deals no damage in the second combat damage step.)

In Alice's third combat phase, **Excruciator** attacks again and is again blocked by r_0 . r_0 's value remains 0. (This behaviour will be modified later in a special case.)

In Bob's untap step, **Vigor** phases back in. In Bob's combat phase, r_0 and **Vigor** attack. r_0 gets blocked by whichever of the twelve **Darksteel Myr** is active, depending on the value of Y ; that one has base power $12Y$.

For each nonzero value of Z , a different **Kangee, Aerie Keeper** adds the value of Z to the power and toughness of **Nomads**, which includes the **Darksteel Myr**, so that its power is $12Y + Z$, which gets added to r_0 's value because of **Vigor**. Note that these **Kangee, Aerie Keepers** are not conditional on anything other than Z ; they will also be used by some other instructions to affect **Nomads**.

If another NumBuild instruction is executed next, Bob's **Vigor** will still be tapped and thus will be unable to block **Dream Fighter**, so it will not phase out and will convert the damage from **Mangara's Equity** into +1/+1 counters on r_0 . (However, **Excruciator**'s damage can't be prevented.) So across the two combat damage steps, instead of r_0 's value being reset to 0, it will be doubled twice, from V to $4V$. In Alice's third combat phase, **Angrath's Marauders** doubles the damage dealt by r_0 , so its value gets tripled in each of the combat damage steps, going from $4V$ to $36V$. Finally, in Bob's combat phase, the value gets doubled twice and gets $12Y + Z$ added, going from $36V$ to $144V + 12Y + Z$.

B.22 3 0 Z ($Z \neq 0$) – JumpFwd Z – Jump forward by Z instructions (=3 Z cards).

- Bob has a 0/1 blue and black **Fiery Emancipation** with added creature types X_3 **Dragon Nomad**, given flying, indestructible and first strike.
- Bob has a 0/1 green and black **Chains of Mephistopheles** given types artifact X_3 **Sliver**, given islandwalk, to which is attached Alice's **Moonsilver Spear** edited to create Scarecrow tokens.
- Alice has a green, blue and red **Reaper King** with added creature types X_3 tR tU .
- Alice has a **Spellbane Centaur** with added creature types X_3 edited to prevent targeting by red sources.
- Alice has a **Shadow Sliver** with added creature types X_3 Y_0 edited to affect **Dragons**.

- Alice has a 0/1 **Melira's Keepers** with added creature types X_3 Sliver tU, given reach, indestructible and lifelink, to which is attached **Spiteful Shadows**.
- Alice has a green and black **Empyrial Archangel** with added creature types X_3 Dragon, with flying removed, given indestructible.
- Alice has a **Lich** given type X_3 .

Recall that the program is a cyclic sequence of cards. To advance through the program, we need to rotate that sequence. The first obstacle is that the sequence isn't all in one place: one of the cards is on the battlefield, while the rest are in Alice's library. Let's fix that.

In Bob's combat phase, **Fiery Emancipation**, and **Chains of Mephistopheles** attack, triggering **Moonsilver Spear**. Alice creates a 4/4 white Scarecrow token with flying. It triggers **Reaper King**, which targets the program permanent; being blue stops it from targeting Bob's **Sterling Grove**, and **Spellbane Centaur** stops it from targeting the Scarecrow token. This destroys the program permanent, sending it to the bottom of Alice's library (because of **Wheel of Sun and Moon**). Next turn, the Scarecrow token will be Alice's target for the **Vaevictis Asmadi, the Dire** that advances the program (because it isn't green, red, or blue), in place of the program permanent. (It will also attack at the same time, but is sacrificed before blockers are declared.)

Now that the program is all in Alice's library, we can rotate it by milling that library – but only one card at a time, otherwise Alice will get to choose the order in which the cards are placed at the bottom of the library. We will do this by making Alice draw cards – the rules force this to be done one at a time (rule 121.2 [5]) – and then using **Chains of Mephistopheles** to replace each card draw with a 1-card mill.

The **Fiery Emancipation** is given shadow by the **Shadow Sliver**, so the **Melira's Keepers** (which also has shadow) blocks it; the tU type stops it from blocking the **Chains of Mephistopheles**.

The **Fiery Emancipation** has power Z (because it's a **Nomad**), and triples its own damage to $3Z$. This damage is dealt with **wither**, but the ability of the **Melira's Keepers** stops it from getting $-1/-1$ counters (but the damage itself is still dealt). The damage triggers **Spiteful Shadows**, which makes the **Melira's Keepers** deal $3Z$ damage to Alice, which the **Empyrial Archangel** redirects to itself. This damage is dealt with lifelink, and Alice's **Lich** replaces the life gain with drawing cards; **Chains of Mephistopheles** replaces each card draw with a 1-card mill, advancing $3Z$ cards through the program.

(The ability counters on the **Melira's Keepers** were placed while its abilities were removed by **Blood Sun**. Alice had a **Platinum Angel** when **Lich**'s abilities were removed, then gained 1 life.)

B.23 3 0 0 – JumpFwd r_0 – Jump forward by r_0 instructions ($=3 \times r_0$ cards).

- Bob has a 0/1 blue and black **Akron Legionnaire** with added creature types X_3 Z_0 Dragon Elf, given flying, indestructible and first strike.

This is a special case of the previous instruction, jumping by r_0 instructions when $Z = 0$.

When $Z = 0$, the **Akron Legionnaire** stops the **Fiery Emancipation** from attacking, and it attacks instead. It is given similar characteristics, but has the creature type **Elf** instead of **Nomad**, thus having power r_0 , which is used as the jump distance instead of Z .

B.24 3 1 Z – JumpBwd Z' – Jump backward by Z' instructions (=3 Z' cards).

As with the previous instruction, if $Z = 0$, Z' is r_0 , and otherwise, Z' is Z . Backward jumps by more than P cards do nothing. P is usually equal to the length of the program in cards; it is the first multiple of the length that is at least 6.

- Alice has a **Joraga Warcaller** with added creature type X_3 , edited to affect Scarecrows, and with $P - 5 +1/+1$ counters on it.
- Alice has a **Justice** with added creature type X_3 , given lifelink and edited to trigger on white creatures.
- Alice has a **Rotlung Reanimator** with type noncopiously set to X_3 , edited to trigger on Scarecrows dying to create white Sand tokens.

In Bob's combat phase, the attacking creatures are the same as before. However, because the **Shadow Sliver** from B.22 is not active, the attacking **Fiery Emancipation** or **Akron Legionnaire** is not given shadow. Instead of being blocked by the **Melira's Keepers**, it is blocked by the Scarecrow token (which has flying).

In the first combat damage step, the **Fiery Emancipation** or **Akron Legionnaire** deals Z' damage with wither to the Scarecrow token. The Scarecrow's power was P (base 4, then boosted by **Reaper King** and this $P - 5$ **Joraga Warcaller**), so after wither damage it's reduced to $P - 3Z'$.

In the second combat damage step, the Scarecrow token deals $P - 3Z'$ damage to the creature it is blocking. **Justice** triggers from that damage, dealing the same amount of damage, redirected to the **Empyrial Archangel**, with lifelink. As before, **Lich** replaces the life gain, and **Chains of Mephistopheles** replaces each card draw, to advance $P - 3Z'$ cards through the program.

If the jump distance exceeds P , the Scarecrow token is killed. **Rotlung Reanimator** triggers, making a white Sand token in its place as the target for **Vaevictis Asmadi, the Dire** next turn.

B.25 3 2 Z – JumpFwdNF Z' – Jump forward by Z' instructions (=3 Z' cards) if the flag is 0/false.**B.26 3 3 Z – JumpBwdNF Z' – Jump backward by Z' instructions (=3 Z' cards) if the flag is 0/false.**

- Alice has a 1/1 **Blinding Angel** with added creature types $X_3 Y_2$.
- Alice has a 1/1 **Blinding Angel** with added creature types $X_3 Y_3$.
- Bob has a 0/3 **Meishin, the Mind Cage** given types $X_3 Y_A$ given reach.
- Alice has a **Shadow Sliver** with added creature types $X_3 Y_2$ edited to affect Dragons.

In Alice's third combat phase, a **Blinding Angel** attacks. If the flag is 0, it deals 1 damage to Bob and triggers its ability to skip Bob's combat phase, so no jump takes place. If the flag is 1, **Meishin, the Mind Cage** reduces its power to 0 and it deals no damage, and the jump takes place as before.

B.27 **3 4 Z – JumpFwdF Z' – Jump forward by Z' instructions**
 (=3Z' cards) if the flag is 1/true.

B.28 **3 5 Z – JumpBwdF Z' – Jump backward by Z' instructions**
 (=3Z' cards) if the flag is 1/true.

- Alice has a 2/2 **Blinding Angel** with added creature types $X_3 Y_4$ with protection from black.
- Alice has a 2/2 **Blinding Angel** with added creature types $X_3 Y_5$ with protection from black.
- Alice has a **Shadow Sliver** with added creature types $X_3 Y_4$ edited to affect Dragons. Again, in Alice's third combat phase, a **Blinding Angel** attacks. This time, if the flag is 1, it is 1/2 and deals 1 damage to Bob and triggers its ability to skip Bob's combat phase, whereas if the flag is 0, it's 2/2 and is not made unblockable by Alice's **Tetsuko Umezawa**, **Fugitive**, and **Meishin**, the **Mind Cage** blocks it (protection from black stops anything else from blocking it).

B.29 **3 6 0 – CallFwd r_0 – Call a function r_0 instructions ahead:**
Jump forward $3 \times r_0$ cards and push $P - 3 \times r_0$ onto the return
stack. If $r_0=0$ or $3 \times r_0 > P$, this is undefined behaviour.

- Bob has a nontoken 3/4 **Progenitor Mimic** copying an **Oubliette** with type overridden to artifact creature with added copiable types Starfish $X_3 Y_6$ tU, with copiable colour blue but noncopiablely coloured green.
- Bob has a **Spellbane Centaur** with added creature types $X_3 Y_6$.
- Bob has a 1/1 **Arwen, Weaver of Hope** with added creature types $X_3 Y_6$ and with $P - 1 +1/+1$ counters on it.
- Alice has a **Shadow Sliver** with added creature types $X_3 Y_6 Z_0$ edited to affect Dragons.
- Alice has a **Wrathful Red Dragon** with added creature types $X_3 Y_6$ tU, with flying removed.
- Alice has a **Willbreaker** with added creature types $X_3 Y_6$.
- The initial return stack consists of a blue 3/4 **Oubliette** with type overridden to artifact creature with added copiable types Starfish $X_3 Y_6$ tU, phasing out an animated **Celestial Convergence** with 1 omen counter on it, both controlled by Alice.

(Note that unlike the other jump instructions, call instructions use only r_0 for the distance, and not the value of Z. This is because it is less common for calls to have a short distance.)

The return stack is made up of **Oubliette** creature tokens under Alice's control. Each one phases out the previous one, so that only the top one is phased in.

In Bob's upkeep, Bob gets a token copy of **Oubliette**, and **Arwen, Weaver of Hope** adds $P +1/+1$ counters to it. Its ability targets the previous top-of-stack token and phases it out; **Spellbane Centaur** stops it from targeting itself or Bob's other blue creatures.

We continue in the same way as a **JumpFwd r_0** instruction, which ends with damage dealt with lifelink to **Empyrial Archangel**, with the life gain replaced by **Lich** and **Chains of Mephistopheles** to advance through the program.

This time, that damage also triggers **Wrathful Red Dragon**, which targets the new **Oubliette** token; the "isn't a Dragon" targeting criterion stops it from targeting Bob's other blue creatures. **Willbreaker** triggers, and Alice gains control of the **Oubliette** token. Then the **Wrathful Red Dragon** trigger resolves, making **Empyrial Archangel** deal $3 \times r_0$ damage with wither to the **Oubliette** token, reducing the number of $+1/+1$ counters on it to

$P - 3 \times r_0$. (Its power is 3 higher than the value pushed onto the return stack, for reasons to be seen later.)

B.30 3 7 Z – Return Z' – Return from a function Z' instructions long: Pop a value S from the return stack, and jump forward $\max(0, S - 3Z')$ cards. If the return stack was empty, end the game in a draw.

- Alice has a **Razorjaw Oni** with added creature types $X_3 Y_7$.
- Alice has a **Spinneret Sliver** with added copiable types $X_3 Y_7$ edited to affect Starfish at timestamp ③.
- Alice has a **Syphon Sliver** with added copiable types $X_3 Y_7$ edited to affect Starfish at timestamp ③.
- Bob has a **Sosuke, Son of Seshiro** with added creature types $X_3 Y_7$ edited to affect and trigger on Dragons.

You may be wondering why this instruction needs the length of the function. It's because we only have relative jumps, as there is no program counter; the program is rotated to advance through it. The call instruction used the offset between the call site and the start of the function, but to return, we need the offset between the end of the function and the call site.

We begin in the same way as a JumpBwd instruction, with a creature with Z' power attacking. **Razorjaw Oni** stops the Scarecrow token from blocking. **Spinneret Sliver** gives reach to the top-of-stack token, timestamped later than the **Blood Sun** (which is currently removing that token's abilities, as it has conditional types X_3 and Y_6), so the top-of-stack token blocks instead. (This is why it is important that the attacking **Fiery Emancipation** or **Akron Legionnaire** is blue rather than green – the top-of-stack token has type $\{U\}$, so it wouldn't be able to block otherwise.)

In the first combat damage step, the attacking **Fiery Emancipation** or **Akron Legionnaire** has 1 extra power from **Sosuke, Son of Seshiro**, so it deals $3Z' + 3$ damage with wither to the top-of-stack token, reducing its power from $S + 3$ to $S - 3Z'$. **Sosuke, Son of Seshiro** triggers to destroy it at end of combat.

In the second combat damage step, the top-of-stack token deals its power in damage if it is still alive. It is given lifelink by **Syphon Sliver**, and the life gain is replaced by **Lich** and **Chains of Mephistopheles** to advance through the program.

If the stack is empty, destroying the base of the stack phases in a **Celestial Convergence**, which triggers in Alice's upkeep to end the game in a draw. This is a convenience feature to allow the same code to be used as a function and as a program.

B.31 3 6 1 – CallBwd r_0 – Call a function r_0 instructions behind: Jump backward $3 \times r_0$ cards and push $3 \times r_0$ onto the return stack. If $3 \times r_0 \geq P$, this is undefined behaviour. May not be used to call a function from within itself.

- Alice has a **Dormant Sliver** with added copiable types $X_3 Y_6$ edited to affect Nomads.
- Bob has a 0/1 blue and black **Skanos Dragonheart** with added copiable types $X_3 Y_6$ Nomad Dragon, edited to affect Rhinos, given flying, indestructible and first strike.

As with the CallFwd instruction, in Bob's upkeep, Bob gets a token copy of **Oubliette**, whose ability targets the previous top-of-stack token and phases it out.

Dormant Sliver stops the **Fiery Emancipation** from attacking, and **Skanos Dragonheart** attacks instead. It is given similar characteristics, and its ability triggers, adding the value of r_0 to its power.

We proceed in the same way as a **JumpBwd** instruction (B.24), up to the **Justice** dealing $P - 3Z'$ damage with lifelink to the **Empyrial Archangel**. Then, in the same way as the **CallFwd** instruction (B.29), **Wrathful Red Dragon** triggers from that damage, and the same sequence of events follows, to give Alice control of the new top-of-stack token and adjust its power.

(**Skanos Dragonheart** doesn't interfere with the **CallFwd** instruction because **Akron Legionnaire** also stops it from attacking when $Z = 0$.)

B.32 3 6 2 – CallBwdR r_0 – Call a function r_0 instructions behind (direct-recursion-capable): Same as CallBwd, but may be used to call a function from within itself.

- Bob has a **Corpsejack Menace** with added creature types $X_3 Y_7 Z_2$.

This variant instruction is necessary when a function calls itself directly: if the regular **CallBwd** instruction is used to do that, the value pushed onto the return stack is the distance between the start of the function and the call site, and on returning, subtracting the length of the function yields a negative value, which results in no jump, which is incorrect.

Corpsejack Menace doubles the number of +1/+1 counters on the new **Oubliette** token to $2P$, and the rest is as before.

B.33 11 Y Z ($Y \neq Z$) – Fless $r_Z r_Y$ – Set the flag to 1 if $r_Z < r_Y$, 0 otherwise. Flag-combining.

- Alice has a **Necrogen Mists** given type X_{11} .
- Bob has a 1/1 **Halfdane** with added creature types $X_{11} tr_Y$, given flying and a +1/+0 counter.
- Bob has a 1/1 **Okk** with added creature types X_{11} Elf, given flying and “Whenever this creature deals combat damage to a player, draw a card.”
- Bob has a 1/1 **Bishop of Binding** given type X_{11} , edited to affect Fractals.
- Alice has a **Tamiyo, Collector of Tales**, with type overridden to artifact creature Fractal.
- Bob has a **Willbreaker** with added creature types X_{11} , and with land type Mountain added at timestamp ④ by an edited **Sealock Monster**.
- Alice has a **Shimmer** choosing Mountain with types $X_2 Y_2$.
- Alice has a **Shimmer** choosing Mountain with types $X_2 Y_3$.
- Alice has a **Shimmer** choosing Mountain with type X_3 .

In Bob's upkeep, **Necrogen Mists** makes Bob discard the flag card if it's in hand, and **Halfdane** triggers and targets r_Y , taking its power and toughness as base power and toughness; its power is now $Y + 1$.

In Bob's combat step, **Halfdane** attacks; **Okk** has $Z + 1$ power, so can attack only if $Y > Z$. If so, it deals its damage to Alice and triggers the added ability to draw a card, which sets the flag to 1.

To accomplish the flag-combining property (using logical OR to combine successive flag-setting instructions), **Bishop of Binding** also attacks; its power is 1, never high enough for it to let **Okk** attack. Its ability targets **Tamiyo, Collector of Tales** (which has been

made a creature and not a planeswalker), triggering **Willbreaker** to make Bob gain control of **Tamiyo, Collector of Tales**. While under Bob's control, it stops Bob from being made to discard the flag card by abilities Alice controls, so the flag remains 1 if it is 1.

This **Willbreaker** has Mountain added to its types (which only works while **Life and Limb** makes it a land). For instructions that end the flag-combining state (those starting with 2 2, 2 3, or 3), **Shimmer** makes it phase out in Bob's untap step if it wasn't already phased out, which ends the duration of **Willbreaker**'s effect and returns control of **Tamiyo, Collector of Tales** to Alice.

B.34 11 Z Z – FlsZero r_Z – Set the flag to 1 if $r_Z=0$, 0 otherwise. Flag-combining.

- Bob has a **Riders of Gavony** with type set to X_{11} and edited to give Rabbit creatures protection from Yetis at timestamp ①.

This is a special case of the previous instruction. Just as with Sub1Cond (B.5), **Riders of Gavony** gives protection from Yetis to all the registers, but only r_Z gets to keep it. If $Y = Z$, this leaves the **Halfdane** in B.33 with no legal targets for its ability, and its base power remains 1, making 1 the value to be compared with.

Appendix C Card Modification Techniques

Here we detail the techniques used while setting up the microcontroller to accomplish the various modifications to cards described in Section 4 and in the instructions (whose implementation is omitted from this paper for space reasons, but can be seen at [10]). We assume Alice has generated an arbitrarily large amount of mana and drawn all the cards she needs using **Dimir Guildmage**. We are able to repeatedly cast the instants and sorceries used below by repeatedly casting **Archaeomancer** and bouncing it with **Capsize**.

- Editing creature types: **Artificial Evolution**
- Editing colour words: **Mind Bend**
- Copiably setting creature type and/or colour and making creatures 1/1: **Croaking Counterpart** combined with **Artificial Evolution** and **Spectral Shift**
- Adding copiable creature types: **Glasspool Mimic** in conjunction with **Artificial Evolution**
- Non-copiablely setting creature type: Use **Blade of Shared Souls** to temporarily make the creature a copy of **Proteus Machine**. Use **Backslide** to turn it face down, then turn it face up and set its creature type.
- Adding non-copiable creature types: **Olivia Voldaren**, modified by **Artificial Evolution** to change Vampire to another type.
- Copiably setting power and toughness, for positive toughness: **Saw in Half** after adjusting as necessary with **Belbe's Armor**, **Enrage**, and/or **Drana, Kalastria Bloodchief**.
- Copiably setting power and toughness to 0/0 and adding type artifact: Have an **Engineered Plague** on Shapeshifter creatures. Cast **Hulking Metamorph** prototyped, and decline to copy anything; it's now 2/2. Cast **Saw in Half** on it, producing token copies that are base 1/1, net 0/0, and then they can copy other creatures while setting base P/T to 0/0. **Grumgully, the Generous** adds a +1/+1 counter to keep it alive.
- Non-copiablely setting power and toughness to the same number: **Gigantoplasm**

- Copiably setting mana value to 0: **Vizier of Many Faces** copiably removes the mana cost, making the mana value 0. (This is usually done so that **Bludgeon Brawl** does not make this give a power boost.) We can repeat this if necessary using **Lithoform Engine** to copy the Embalm ability, untapped by **Twiddle**.
- Copiably setting card type to (only) artifact: **Imposter Mech**, after targeting the original with **Donate**.
- Adding type artifact: **Memnarch**
- Copiably adding type creature: **Urza, Prince of Kroog**, in conjunction with **Memnarch** if necessary, and with the creature type edited by **Artificial Evolution**.
- Copiably adding flying: **Irenicus's Vile Duplication** copiably adds flying.
- Adding keyword ability counters – flying, indestructible, reach, first strike, double strike, lifelink, deathtouch, vigilance, trample: **Kathril, Aspect Warper**, having used **Dimir Guildmage** to discard other cards used in the construction: **Healer's Flock**; **Darksteel Myr**; **Halana and Alena, Partners**; **Sylvia Brightspear**; **Questing Beast**; and **Quartzwood Crasher**. **Regrowth** gets back the discarded cards afterwards.
- Adding protection from a colour: Have a **Council Guardian** enter the battlefield, and use **Ballot Broker** to make sure the right colour wins the vote. Then use **True Polymorph** to turn it into what it should be.
- Adding islandwalk: **Fishliver Oil**, copied with **Mythos of Illuna**
- Adding “Whenever this creature deals combat damage to a player, draw a card.”: Use **Blade of Shared Souls** to temporarily make the creature a copy of an **Ascendant Spirit**, which itself is copiably an Angel by **Glasspool Mimic**+**Artificial Evolution**, and then activate its last ability to add this ability. Generate the snow mana with a Snow-Covered land repeatedly untapped by **Twiddle**.
- Removing flying: Use **Blade of Shared Souls** to temporarily make the creature a copy of **Mist Dragon**, and use its ability to remove flying.
- Adding +1/+1 counters: **Kathril, Aspect Warper** and **Resourceful Defense**
- Adding -1/-0 counters: **Jabari's Influence** to get the first one, and multiple copies of **Resourceful Defense** to get more.
- Adding +1/+0 counters: **Dwarven Armorer** produces +1/+0 counters. Multiple copies of **Resourceful Defense** multiply the counters and then move them, as necessary.
- Creating arbitrary tokens: **Rotlung Reanimator** with **Artificial Evolution** creates tokens of arbitrary types, and **Saw in Half** sets their sizes after adjusting with **Belbe's Armor**.
- Setting colours: **Prismatic Lace**
- Giving Bob control of cards: **Donate**

Example: We make each **Vaevictis Asmadi, the Dire** into a 1/1 Sliver Beast Reflection by casting **Croaking Counterpart** targeting a real **Vaevictis Asmadi, the Dire**, then responding by casting **Artificial Evolution** targeting **Croaking Counterpart** to change Frog to Sliver. Once the 1/1 Sliver is present, we return **Artificial Evolution** to our hand using **Archaeomancer**. We cast **Glasspool Mimic**; edit it with **Artificial Evolution** replacing Shapeshifter with Beast; cast **Capsize** with buyback on the **Archaeomancer**; recast **Archaeomancer** using **Leyline of Anticipation** to get back **Artificial Evolution** again; then use **Artificial Evolution** one more time to replace Rogue with Reflection. Then **Glasspool Mimic** resolves and becomes a 1/1 Sliver Beast Reflection copy of **Vaevictis Asmadi, the Dire**. We can create further token copies of this with **Mythos of Illuna** (all the creature type additions are copiable), and then use **Capsize** to return the original **Glasspool Mimic** to our hand for the next time we need to use it.

Appendix D Decklist

Table 5 on the facing page contains a decklist suitable for bringing to a Legacy tournament which could set up the microcontroller. Adjust the number of basic lands according to the program you wish to write; the following decklist contains enough to write the Collatz sample program. You can also leave out cards that are only used in instructions that aren't present in the program you wish to write. E.g. **Oubliette** is only used in Call and Return instructions.

■ **Table 5** Decklist to play the *Magic* Microcontroller in a Legacy tournament

| Card | Purpose | Card | Purpose |
|-----------------------------|----------------------------|-------------------------------|-----------------------|
| 4 Ancient Tomb | Bootstrap | 4 Lotus Petal | Bootstrap |
| 4 Grim Monolith | Infinite mana | 4 Power Artifact | Infinite mana |
| 4 Gemstone Array | Infinite mana | 4 Dimir Guildmage | Draw rest of deck |
| 13 Plains | Program | 13 Snow-Covered Plains | Program |
| 13 Island | Program | 18 Snow-Covered Island | Program |
| 13 Swamp | Program | 13 Snow-Covered Swamp | Program |
| 18 Mountain | Program | 8 Snow-Covered Mountain | Program |
| 18 Forest | Program | 10 Snow-Covered Forest | Program |
| 13 Wastes | Program | 10 Snow-Covered Wastes | Program |
| 1 Memnarch | Make token copies | 1 Mythos of Illuna | Make token copies |
| 1 Capsize | Set up | 1 Archaeomancer | Set up |
| 1 Artificial Evolution | Edit cards | 1 Mind Bend | Edit cards |
| 1 Prismatic Lace | Edit cards | 1 Spectral Shift | Edit cards |
| 1 Glasspool Mimic | Add types | 1 Olivia Voldaren | Add types |
| 1 Proteus Machine | Set creature types | 1 Backslide | Set creature types |
| 1 Argent Mutation | Set up | 1 Leyline of Anticipation | Set up |
| 1 Gigantoplasm | Edit power/toughness | 1 Croaking Counterpart | Set up |
| 1 Saw in Half | Edit power/toughness | 1 Belbe's Armor | Edit power/toughness |
| 1 Enrage | Edit power/toughness | 1 Drana, Kalastria Bloodchief | Edit power/toughness |
| 1 Engineered Plague | Edit power/toughness | 1 Grungully, the Generous | Edit power/toughness |
| 1 Hulking Metamorph | Edit power/toughness | 1 Vizier of Many Faces | Edit mana value |
| 1 Lithoform Engine | Edit mana value | 1 Twiddle | Edit mana value |
| 1 Imposter Mech | Edit types | 1 Donate | Edit control |
| 1 Astral Dragon | Add types and abilities | 1 Irenicus's Vile Duplication | Add abilities |
| 1 Urza, Prince of Kroog | Add types | 1 Kathril, Aspect Warper | Add abilities |
| 1 Council Guardian | Add abilities | 1 Ballot Broker | Add abilities |
| 1 True Polymorph | Add abilities | 1 Fishliver Oil | Add abilities |
| 1 Blade of Shared Souls | Change types and abilities | 1 Ascendant Spirit | Add abilities |
| 1 Mist Dragon | Remove abilities | 1 Resourceful Defense | Add counters |
| 1 Jabari's Influence | Add counters | 1 Fishliver Oil | Add abilities |
| 1 Dwarven Armorer | Add counters | 1 Sealock Monster | Add types |
| 1 True Polymorph | Add abilities | 1 Grand Melee | Control combat |
| 1 Stormtide Leviathan | Control combat | 1 Tetsuko Umezawa, Fugitive | Control combat |
| 1 Worship | Control combat | 1 VaeVictis Asmadi, the Dire | Advance program |
| 1 Wheel of Sun and Moon | Advance program | 1 Tajuru Preserver | Advance program |
| 1 Dralnu's Crusade | Conditional mechanism | 1 Life and Limb | Conditional mechanism |
| 1 Blood Sun | Conditional mechanism | 1 Strata Scythe | Conditional mechanism |
| 1 Healer's Flock | Keyword abilities | 1 Dream Fighter | Conditional mechanism |
| 1 Shadow Sliver | Conditional mechanism | 1 Bludgeon Brawl | Conditional mechanism |
| 1 Bloodthirster | Advance program | 1 Masked Gorgon | Constrain targets |
| 1 Spectral Guardian | Constrain targets | 1 Sterling Grove | Constrain targets |
| 1 Ivory Mask | Constrain targets | 1 Illusionary Terrain | Control types |
| 1 Joraga Warcaller | Registers | 1 Riders of Gavony | Registers |
| 1 Nevermore | Control choices | 1 Aggressive Mining | Control choices |
| 1 Root Maze | Control choices | 1 Tomorrow, Azami's Familiar | Flag |
| 1 Chokey | Control choices | 1 Suppression Bonds | Control choices |
| 1 Stony Silence | Control choices | 1 Cursed Totem | Control choices |
| 1 Recycle | Control state | 1 Mirror Gallery | Control state |
| 1 Corrosive Mentor | Add abilities | 1 Halana and Alena, Partners | Instructions |
| 1 Archpriest of Iona | Instructions | 1 Shape Stealer | Instructions |
| 1 Necrogen Mists | Instructions | 1 Halfdane | Instructions |
| 1 Tanuki Transplanter | Instructions | 1 Furtive Homunculus | Instructions |
| 1 Omnath, Locus of Mana | Instructions | 1 Wrathful Red Dragon | Instructions |
| 1 Belligerent Brontodon | Instructions | 1 Smog Elemental | Instructions |
| 1 Hornet Nest | Instructions | 1 Phantom Steed | Instructions |
| 1 War Elemental | Instructions | 1 Tocatli Honor Guard | Instructions |
| 1 Ward Sliver | Instructions | 1 Godhead of Awe | Instructions |
| 1 Wandering Wolf | Instructions | 1 Behind the Scenes | Instructions |
| 1 Spinneret Sliver | Instructions | 1 Quartzwood Crasher | Instructions |
| 1 Arwen, Weaver of Hope | Instructions | 1 Aether Flash | Instructions |
| 1 Charisma | Instructions | 1 Skeleton Key | Instructions |
| 1 Serpent of Yawning Depths | Instructions | 1 Field Marshal | Instructions |
| 1 Questing Beast | Instructions | 1 Vigor | Instructions |
| 1 Toralf, God of Fury | Instructions | 1 Gideon's Intervention | Instructions |
| 1 Progenitor Mimic | Instructions | 1 Volcano Hellion | Instructions |
| 1 Artificer Class | Instructions | 1 Syr Elenora, the Discerning | Instructions |
| 1 Slithering Shade | Instructions | 1 Suntail Hawk | Instructions |
| 1 Strength-Testing Hammer | Instructions | 1 Ancient Gold Dragon | Instructions |
| 1 Rat Colony | Instructions | 1 Goblin Pyromancer | Instructions |
| 1 Celestial Convergence | Instructions | 1 Alert Heedbonder | Instructions |
| 1 Spiritual Sanctuary | Instructions | 1 Abyssal Specter | Instructions |
| 1 Catacomb Dragon | Instructions | 1 Taii Wakeen, Perfect Shot | Instructions |
| 1 Excruciator | Instructions | 1 Aegar, the Freezing Flame | Instructions |
| 1 Khorvath Brightflame | Instructions | 1 Sylvia Brightspear | Instructions |
| 1 Mangara's Equity | Instructions | 1 Darksteel Myr | Instructions |
| 1 Ojutai, Soul of Winter | Instructions | 1 Angrath's Marauders | Instructions |
| 1 Kangee, Aerie Keeper | Instructions | 1 Shimmer | Instructions |
| 1 Fiery Emancipation | Instructions | 1 Chains of Mephistopheles | Instructions |
| 1 Captain's Claws | Instructions | 1 Steely Resolve | Instructions |
| 1 Reaper King | Instructions | 1 Akron Legionnaire | Instructions |
| 1 Discordant Spirit | Instructions | 1 Blinding Angel | Instructions |
| 1 Meishin, the Mind Cage | Instructions | 1 Bower Passage | Instructions |
| 1 Darksteel Myr | Instructions | 1 Moonsilver Spear | Instructions |
| 1 Spellbane Centaur | Instructions | 1 Melira's Keepers | Instructions |
| 1 Spiteful Shadows | Instructions | 1 Empyrial Archangel | Instructions |
| 1 Lich | Instructions | 1 Justice | Instructions |
| 1 Rotlung Reanimator | Instructions | 1 Oubliette | Instructions |
| 1 Willbreaker | Instructions | 1 Razorjaw Oni | Instructions |
| 1 Sosuke, Son of Seshiro | Instructions | 1 Syphon Sliver | Instructions |
| 1 Dormant Sliver | Instructions | 1 Skanos Dragonheart | Instructions |
| 1 Corpsejack Menace | Instructions | 1 Okk | Instructions |
| 1 Bishop of Binding | Instructions | 1 Tamiyo, Collector of Tales | Instructions |
| 1 Shimmer | Instructions | 1 Sporemound | Instructions |
| 1 Polyraptor | Instructions | 1 Chief of the Scale | Instructions |
| 1 Gruul Ragebeast | Instructions | 1 Sliver Hivelord | Instructions |